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Modélisation Probabiliste de Dégradations Multiphasiques pour l'Optimisation de la Maintenance d'Infrastructures en Génie Civil :

Application à une structure en béton armé immergée

JURY

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Preface

From November 2012 to November 2015, I was a PhD candidate at the University of Nantes. I worked on the Ecole des Mines de Nantes campus. It was a very pleasant experience, and this is not just because of the interesting subject of research. Sometimes that is not all. In this preface, I want to thank all the people who I was lucky enough to share this period of time with and who made it the experience it was, it is.

Franck, I'm honoured and glad to have had you as the director of the PhD committee, you have shown me what it is like to be a hardworking researcher and a dynamic director, while remembering the occasional laugh.

Bruno, at the start of this thesis you set the rules for having a beer together, finally that day came! Thank you very much for all the things you said, done and advised me during the last three years. Cheers!

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Members of the jury, thank you for your time and for your presence in the defence of this thesis. It was a real pleasure to meet you all and to defend my work in your presence.

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My aunt, Nazira. Her knowledge in bridges is impeccable. Thank you. And to your son Tony "Captain", good luck for your PhD in Brest.

Petra, what was supposed to be stressful and hard, you turned into droll and memorable, and most importantly, you are an amazing cook! Thank you for your help in the writing of this manuscript, both on and off the screen. I wish you courage for your PhD in Rouen.

To my family, no words can express my gratitude for your sacrifices and continuous support.

Naoum, my father. Thank you for being an infinite source of love, and advices for the last four years, and the twenty two before them.

My mother, Gilberte. On behalf of everyone that was present the day of my defence, thank you for the delicious food! Merci mama a’ kela chi. Bhebik ktir.

Nagi, my brother. All words that can possibly be said here, are in this word. And you took it to another level.

This thesis is dedicated to Tante Laurisse.

You are the bravest woman I know.

You will always remain in our hearts.

Nantes, 23/11//2015

Boutros Naoum EL HAJJ

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Notations

Some of the concepts used in this thesis can be interpreted in several ways. Below we define our interpretation of the concepts.

Corrective Maintenance	Maintenance performed after failure and aims to restore a property in a state in which it can perform a required function.
Degradation	The falling from a higher to a lower level in the condition of a system. More degradation implies a worse condition level.
Imperfect maintenance	Maintenance actions that does not restore the condition of the system to as good as new.
Inspection	The process of measuring, examining, testing, gauging, or otherwise detecting any deviations from specifications.
Gamma process	A gamma process is a stochastic process with independent gamma distributed increments.
Maintenance	The combination of all technical and associated actions by which a system is kept in, or restored to, a state in which it can perform its designated functions.
Maintenance optimization	The process that attempts to find those maintenance and inspection times and techniques where some decision criteria (costs and performance in example) are optimized.
Measurable degradation	Degradation that can be expressed through a continuous measurable quantity for which intervention limits can be set.
Meta-model	A degradation model used in maintenance management function of a small number of parameters, based on the probabilistic pertinence and physical expertise on one hand, and indicators of degradation and durability directly accessible from NDT on the other hand.
Monitoring	The continuous observation over time of the condition of the structure for any changes, which may occur.
Multiphasic pathologies	A pathology having many phases or stages, where each phase or stage is governed by a different physical mechanism that can be characterized using an appropriate physical law and indicators.

Non-Destructive Testing	Inspection of components using equipment to reveal the defects, such as magnetic particles or ultrasonic methods without harming the structure
Preventive Maintenance	Maintenance performed at predetermined intervals or according to prescribed criteria and aims to reduce the probability of failure or the degradation of the functioning of a structure.
Reliability	Likelihood of a structure or component to fulfil its functions during a given period.
Risk	Risk is a measure of possible loss or injury, and is expressed as the combination of the incident probability and its consequences.
State-dependent gamma process	A non-stationary gamma process with a state-dependent distributed increments.
Stationary process	Stochastic process with identically distributed increments.

Abbreviations

CBM	Condition Based Maintenance
CM	Corrective Maintenance
COV	Coefficient of Variation
DT	Destructive Testing
EM	Estimation Maximisation
GP	Gamma Process
MLE	Maximum Likelihood Estimation
MM	Meta Model
MSE	Mean Squared Error
NDT	Non Destructive Testing
NHPP	Non-Homogeneous Poisson process
PDF	Probability Density Function
PM	Preventive Maintenance
RBI	Risk Based Inspection
SDGP	State Dependent Gamma Process
SEM	Stochastic Estimation Maximization
SHM	Structural Health Monitoring
TBM	Time Based Maintenance
c.f.	Short for the Latin *word confer* — which means *compare*
e.g.	Short for the Latin phrase *exempli gratia* — which means *for example*
etc.	Short for the Latin *et cetera* — which means *and so on*
i.e.	Short for the Latin *id est* — which means *that is*, *namely*, or *in other words*

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Summary in French

Résumé en Français

– Abstract –

Notre société est face à des enjeux importants en termes de génie civil et de maintenance avec un grand nombre de structures vieillissantes (digues, structures portuaires, ouvrages d'art, …) et des challenges liées à la construction de nouveaux ouvrages dans des environnements plus agressifs et moins accessibles (systèmes liées aux Energies Marines Renouvelables, par exemple) avec des forts enjeux de rentabilité et donc de productivité. Dans la recherche de la durabilité, de la sécurité, de la disponibilité ainsi que de l'acceptation sociétale, il devient primordial de s'assurer de l'intégrité de l'ouvrage et de la minimisation des conséquences des défaillances. Ceci peut se traduire par l'assurance de livraison de produits plus fiables mais aussi par l'amélioration des stratégies de maintenance et de suivi de ceux-ci tout au long de leur cycle de vie. Aujourd'hui, pour en améliorer ses performances, la maintenance est conditionnelle, dépendante des résultats de l'instrumentation in-situ et de contrôles ponctuels. Son organisation ou optimisation doit notamment conduire à inspecter au bon moment, au bon endroit et avec la meilleure technique dans un contexte de budget limité. Cette recherche d'amélioration de performance dans un cadre fortement incertain se traduit par plusieurs volets avec, sans recherche d'exhaustivité sur ces volets :

- *l'amélioration des techniques d'auscultation des ouvrages ;*
- *l'évolution des modèles de dégradation devant assurer un meilleur lien entre la connaissance sur les pathologies étudiées, la nature de l'information collectée que nous pouvons par exemple rapprocher des techniques de contrôles non destructifs, un caractère prédictif nécessaire pour l'intégration dans des critères décisionnels complexes et tout ceci dans un cadre fortement incertain.*

Plus explicitement, on peut considérer que les techniques de contrôles non destructifs (CND) offrent des potentiels de tout premier plan mais sous exploités. Par ailleurs les incertitudes concernant le matériau, l'environnement et la mesure CND ne sont pas prises en compte. Ce constat peut partiellement être lié aux approches développées par une grande majorité de chercheurs de la communauté scientifique : de notre point de vue, ils cherchent à relier de manière très fine l'expertise approfondie des phénomènes de

dégradation traduite par un ensemble de modèles physiques définis dans des contextes supposés homogènes à des observations obtenues par CND de structures plus ou moins hétérogènes. Nous rapprocherons cette approche des stratégies « bottom-up » qui cherchent à définir un comportement global d'un ouvrage par agrégations successives de phénomènes locaux observés. Alors que ces modèles ont largement prouvé leur intérêt pour représenter des tendances ou expliquer des phénomènes, ils montrent leurs limites en phase d'exploitation des structures, confrontés à une limite de mesure de leurs paramètres. La thèse vise à proposer une nouvelle approche de modélisation de type méta-modèle pour la stratégie de maintenance pour les structures dégradées à partir de résultats CND. On entend par méta-modèles des modèles à faible nombre de paramètres reposant sur l'expertise physique et la pertinence probabiliste d'une part et sur les indicateurs de dégradation et de durabilité directement accessibles à partir de contrôles non destructifs CND d'autre part. La thèse propose une modélisation de phénomènes multi-phasiques de dégradation du béton armé reposant sur des processus stochastiques non-stationnaires dépendant de l'état.

Introduction et problématique

La maintenance de structures et infrastructures est une question sensible pour toute la société, principalement parce qu'elle consomme de plus en plus de ressources financières et naturelles qui de nos jours deviennent limitées. Ceci s'explique par le nombre croissant de structures à gérer, par l'âge des infrastructures existantes et par les exigences de plus en plus fortes en termes de sécurité et de performance (augmentation du trafic).

En outre, étant données les conséquences et les répercussions d'une défaillance sur l'homme, sur l'économie et sur l'environnement, la maintenance est à considérer comme une composante à part entière à intégrer dès la phase de conception pour toute nouvelle structure, et comme indispensable pour les structures anciennes à fort enjeu: ponts, hôpitaux, aéroports, ports et digues. La surveillance de ces structures doit être effectuée tout au long de leur durée de vie, principalement pour signaler les défauts précurseurs de la défaillance, mais également pour suivre les performances de la structure que l'on cherchera à quantifier pour une meilleure compréhension des pathologies et calibration des modèles.

Le standard européen (EN 13306, 2001) définit la maintenance comme « *la combinaison de toutes les actions techniques, administratives et de gestion par lesquelles un système est maintenu, ou restitué à un état dans lequel il peut exercer ses fonctions désignées* ». En d'autres termes, ils existent de nombreuses combinaisons possibles de ces actions –techniques d'inspection et méthodes de réparations, par exemples – renforçant ainsi la nécessité de la construction d'un modèle d'optimisation pour la planification et la sélection de ces actions.

En visant une maintenance optimisée, on argumente que le choix d'une opération de maintenance doit être déclenché par l'état du système lui-même plutôt que périodiquement. Autrement dit, la maintenance est conditionnelle, c'est-à-dire basée sur l'état de la structure. En fait, si chaque action est déclenchée au meilleur moment pour la structure, une politique de maintenance optimale sera effectivement accessible (dans un contexte de contraintes prédéfinies, par exemple performance et coût). Cette approche est connue sous le nom anglais de *Condition-Based Maintenance* (*CBM*), ou maintenance basée sur l'état ; elle vise à examiner la structure au bon moment et avec la meilleure technique disponible pour sélectionner la meilleure décision parmi l'ensemble des actions possibles. L'état d'une structure est évalué en termes de risque associé aux niveaux de dégradation qui sont généralement exprimés sur la base d'indicateurs de dégradation. Ainsi, les résultats de l'évaluation des risques constituent l'entrée de la prise de décision.

Ces indicateurs de dégradation peuvent être mesurés par des inspections, ou estimés par l'intermédiaire de modèles dégradation. Ces estimations sont classiquement effectuées sur la base de modèles physiques. Cependant, l'actualisation des modèles reste nécessaire pour renforcer le caractère prédictif du modèle. Elle est réalisée au moyen d'évaluations de l'état courant de la structure par le biais d'informations liées au CND ; cette étape reste clairement un challenge puisque les sorties CND sont des informations physiques très indirectement liées aux entrées des modèles de dégradation.

Par la suite, on montrera que de nouvelles approches permettent de résoudre cette difficulté. Il s'agit de modéliser la dégradation en dépassant les approches classiques, en particulier en termes d'intégration des mesures CND et des incertitudes de prédiction. Pour cet objectif, dans cette thèse, nous analysons les techniques de modélisation de dégradations classiques (spécialement les modèles probabilistes) utilisés pour les problèmes de maintenance dans le but de souligner les caractéristiques qui définissent un « bon » modèle de dégradation, c'est à dire satisfaisant un certain nombre de critères que nous énumérerons.

Une analyse bibliographique sur les modèles de dégradation dans un contexte fiabiliste et d'optimisation de la maintenance permet d'identifier deux grandes tendances : d'une part les modèles riches basés sur l'explicitation de la physique de dégradation (§ 2.4.3), et de l'autre des modèles fréquentielles reposant sur la construction de quantités statistiques (§ 2.4.2) (Frangopol *et al*.,2004). Ainsi, la qualité d'un modèle de dégradation ne se limite plus à la modélisation de la pathologie et à sa compatibilité aux données et aux connaissances disponibles mais aussi à sa qualité de prédiction de la dégradation, sa facilité à intégrer de nouvelles observations notamment par CND, et enfin sa commodité d'implémentation dans des méthodes d'optimisation de la maintenance de plus en plus complexe.

On assiste depuis la fin des années 1990 à une complexification croissante des modèles de dégradation, intégrant de plus en plus de couplages physico-chimiques et mécaniques, conduisant à une explosion du nombre de paramètres : il est courant par exemple de nos jours de disposer d'un modèle de propagation d'ions chlorures de plus de 10 paramètres alors que le modèle type des années 90 en comportait 2. L'utilisation de ces avancées dans un contexte fiabiliste pose plusieurs difficultés : comment « probabiliser » des modèles dont les paramètres sont généralement des variables aléatoires corrélées mais dont on ne connaît aucune information a priori et comment réaliser des études de sensibilité en l'absence de ces tendances ? Plus le nombre de paramètres augmente, plus, dans un cadre stochastique notamment, le nombre d'expériences nécessaires à paramètres contrôlés augmente, en fonction puissance. *On assiste* *alors à une première rupture : la calibration ou l'identification probabiliste*. Dans un contexte d'inspection-maintenance, il faut ajouter à cela que les techniques CND permettant le diagnostic et l'évaluation des propriétés des matériaux et de leurs pathologies font face à des défis de plus en plus difficiles et à une évolution lente des performances. De plus, le phénomène physique mesuré par ces techniques est souvent sensible à des effets combinés (porosité-teneur en eau pour le béton notamment) qu'il faut dé-corréler pour alimenter les modèles complexes, rendant très délicates les évaluations des corrélations : elles sont issues d'une même technique CND et le fruit d'un découplage numérique, donc sensible à la formulation même du modèle et de ses a priori. On assiste à une seconde rupture tenant au fossé qui se creuse entre les vitesses de développement des modèles de dégradation à niveaux de complexité élevés et les vitesses de développements nécessitant de lever des verrous technologiques mais aussi numériques (inversion) pour les techniques CND : *la seconde rupture se situe alors entre les paramètres modèles et les observables CND.*

Notre objectif est de modéliser l'évolution de la dégradation directement partir de l'information disponible. Cette approche nous conduit à analyser l'évolution des observables. Cependant, sans la prise en compte des propriétés de dégradation, une telle approche conduirait à la construction de modèles purement statistiques nécessitant un très grand nombre d'observations pour capturer les effets mécanistes des processus de dégradation associés.

Une approche qui semble prometteuse pour la maintenance dans le génie civil repose sur des processus stochastiques, notamment le processus gamma (Van Noortwijk 2009). Elle permet de notre point de vue d'aborder la modélisation de la dégradation par le biais d'observations issues notamment de contrôles non destructifs tout en conservant les aspects les plus importants de la physique dans le modèle et une facilité d'intégration dans des critères de décision de maintenance complexes. On peut cependant souligner les difficultés de modélisation lorsque les pathologies retenues présentent des comportements non stationnaires dans le temps (effets d'accélération ou décélération de la dégradation). Des extensions appelées modèles conditionnels ou dépendants de l'état (Vatn 2012) et (Zouch *et al*., 2011) permettent de modéliser ces effets non stationnaires uniquement en fonction des niveaux de dégradation. Par contre on peut noter dans la construction de leur modèle un manque de procédure robuste d'identification des paramètres d'entrée ainsi qu'un manque de caractère applicatif, limites rendant délicates leur appropriation et la validation dans des contextes opératoires (Riahi *et al*., 2010).

Ainsi, dans cette thèse, nous proposons une nouvelle formulation des processus stochastiques pour la modélisation de dégradation non-stationnaire qui permet une meilleure intégration des données de mesures issues de CND. Par les propriétés mathématiques des processus stochastiques, nous visons également à diminuer le temps de calcul dans un contexte de maintenance. Cette formulation de processus stochastique est appelée : « méta-modèles ».

On entend par méta-modèles des modèles à faible nombre de paramètres reposant sur l'expertise physique et la pertinence probabiliste d'une part et sur les indicateurs de dégradation et de durabilité directement accessibles à partir de contrôles non destructifs CND d'autre part. Ils ont donc la particularité d'avoir en entrée à la fois des indicateurs de dégradations et des indicateurs de durabilité, les deux étant présentés sous forme de processus stochastiques.

Un des objectifs de cette thèse est de promouvoir l'utilisation des méta-modèles et de montrer leur applicabilité pour des politiques de maintenance conditionnelle appliquées aux structures et infrastructures, pour lesquelles les données peuvent être accessibles par CND.

Un challenge de cette étude est de rejoindre deux communautés scientifiques qui traitent de l'optimisation de la maintenance: d'une part, une communauté intéressée par le développement des modèles de dégradation probabilistes, et d'autre part, une communauté traitant les problèmes mathématiques d'optimisation.

Dans cette thèse, nous étudions des dégradations cumulatives qui sont généralement rencontrées dans les infrastructures de génie civil. Les cas plus rares comme l'autoréparation ne sont pas étudiés. De plus, nous considérons un cadre discret en temps pour prendre des décisions et des inspections, ce qui est suffisant au regard des fréquences d'inspection et de maintenance.

Résultats

Dans cette thèse nous répondrons à un ensemble de questions en termes de maintenance des structures et des infrastructures. Les challenges de maintenance sont nombreux, cependant, dans ce manuscrit, nous limitons les discussions à un ensemble résumé par les questions-réponses suivantes :

Comment capturer, modéliser et prévoir au mieux la dégradation pour répondre aux nouvelles nécessités de la maintenance ?

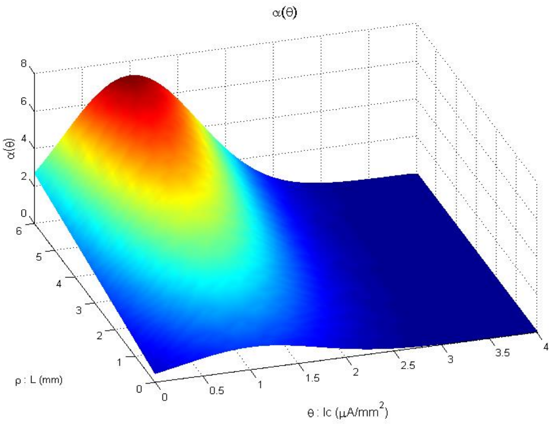
Cette question a été analysée en deux parties: la première partie concernant les nouvelles nécessités de la maintenance, et la deuxième partie relative aux caractéristiques nécessaires dans un modèle de dégradation pour pouvoir répondre à ces nécessités.

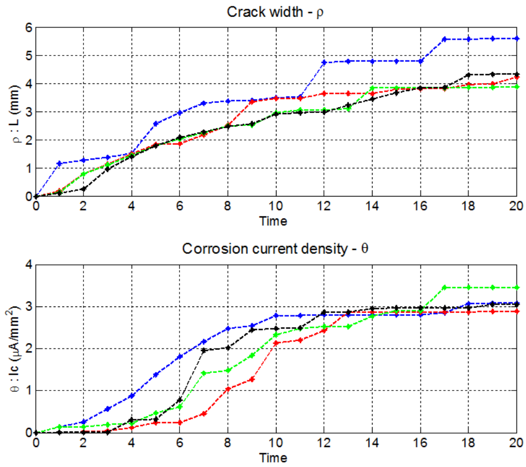
Tout d'abord, il faut signaler que les éléments de maintenance ont beaucoup évolué au cours des deux dernières décennies. Dans le chapitre 2, une bibliographie sur les inspections (§ 2.3), les systèmes de gestion de la maintenance (§ 2.2) et la décision (§ 2.3.4), a montré la nécessité d'étendre les caractéristiques des modèles de dégradation (§ 2.4).

Le rôle des modèles de dégradation doit évoluer afin de capturer et simuler la physique de la dégradation, prédire de façon fiable cette dégradation en utilisant toutes les informations disponibles, prendre en compte les incertitudes, intégrer des données CND, et enfin, permettre leur intégration dans les plates-formes complexes de maintenance conditionnelle (§ 2.4.1).

En génie civil, les processus de dégradation sont généralement multiphasiques, multivariés et non-stationnaires. Dans cette thèse, nous avons construit un méta-modèle qui a répondu avec efficacité à ces préoccupations. Le défi multiphasique (§ 4.2) a été résolu en proposant une approche uniforme pour modéliser la dégradation (§ 3.3.1), où, pour chaque phase, nous examinons et choisissons les indicateurs de dégradation appropriés (§ 3.2.1), puis nous les modélisons en utilisant des processus stochastiques dépendant de l'état (par exemple, §3.3.2). Deuxièmement, dans cette thèse, nous nous sommes intéressés aux modèles de dégradation multivariés où la non-stationnarité a été modélisée en utilisant des processus de gamma dépendant de l'état bi-variés continus (§ 2.5.3) pour leurs avantages mathématiques (§ 2.4.4). La construction du modèle de dégradation dépendant de l'état a été discutée dans le cas de corrosion par chlorure d'une structure en béton armé (§ 3.2).

Pour conclure, le méta-modèle vise à modéliser une pathologie de la dégradation avec une vision globale pour relever les défis en termes de maintenance de la structure. Il permet alors de guider le choix des indicateurs physiques pour répondre à la fois à la caractérisation de chaque phase, et l'assurance d'une continuité en termes d'évaluation de la dégradation à travers le cycle de vie de la structure.

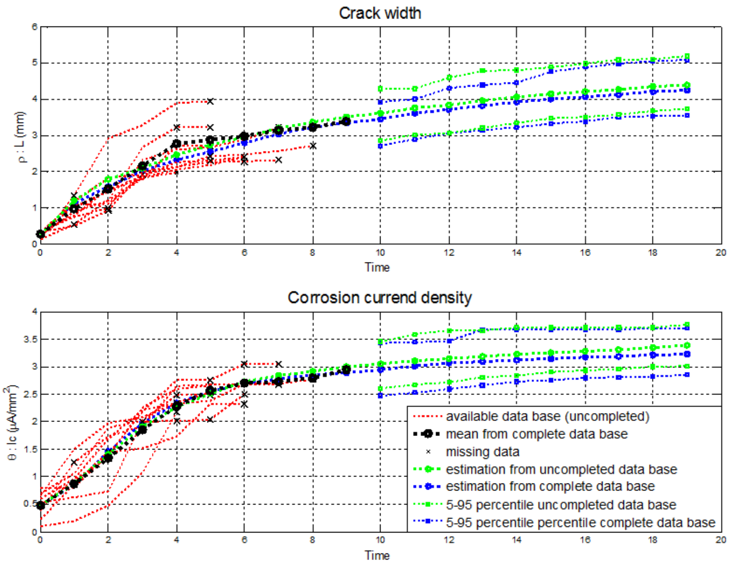




Dans la figure à gauche (*cf.*, Figure 3.5), on illustre une des deux fonctions de formes du processus gamma qui est dépendante de l'état par le biais des indicateurs de dégradation (ρ *et* θ). Dans la figure à droite (*cf.*, Figure 3.7), on illustre 4 trajectoires simulées en utilisant le processus gamma non-stationnaire bi-varié dépendant de l'état.

Comment être réaliste ? Comment réagir face aux inspections non informatives et aux informations manquantes ?

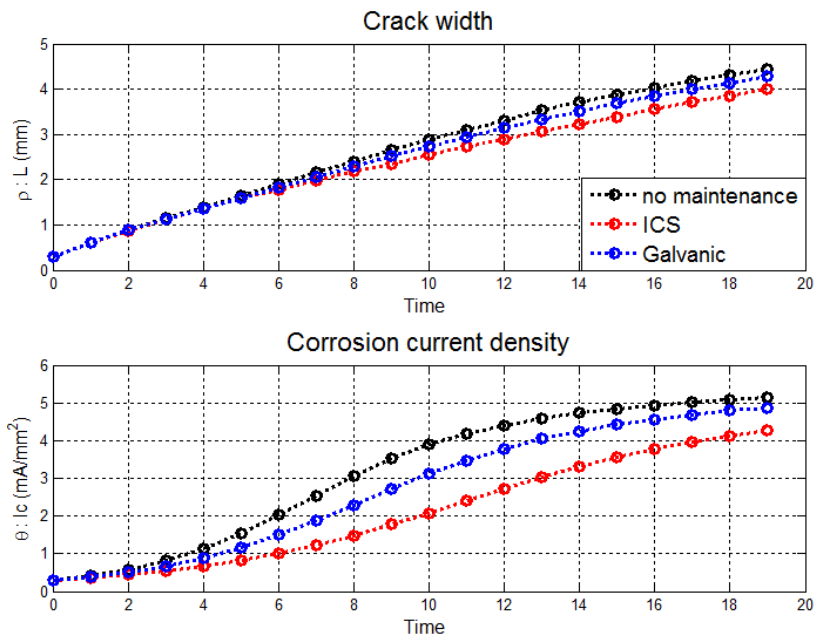
Les incomplétudes et irrégularités de bases de données et des inspections sont des aspects inséparables de toute situation réaliste en génie civil (§ 2.3.3). Nous avons testé le modèle dans des situations avec données manquantes, censurées ou tronquées (§ 3.3.3.2 et § 3.4.3), et nous avons développé une solution reposant sur l'extension de la méthode d'estimation classique du maximum de vraisemblance ou MLE (§ 3.3.3.1), reposant sur l'algorithme Stochastique Estimation-Maximisation ou SEM (§ 3.3.3.2). En outre, nous avons observé dans le cas où des bases de données hétérogènes ont été incluses dans le processus d'estimation (§ 2.3.3.2) que le processus peut être remarquablement amélioré (§ 3.4.2). Ces résultats décrivent la capacité du modèle à répondre à des situations réalistes.



Dans cette figure (*cf.*, Figure 3.11), on illustre l'application de l'algorithme SEM sur une base de données qui souffre de données manquantes et censurées à droite (en rouge) pour estimer et prédire la dégradation (en vert) en comparant avec une prédiction faite en utilisant une base de donnée complète (en bleu).

Comment modéliser l'effet d'une action de maintenance?

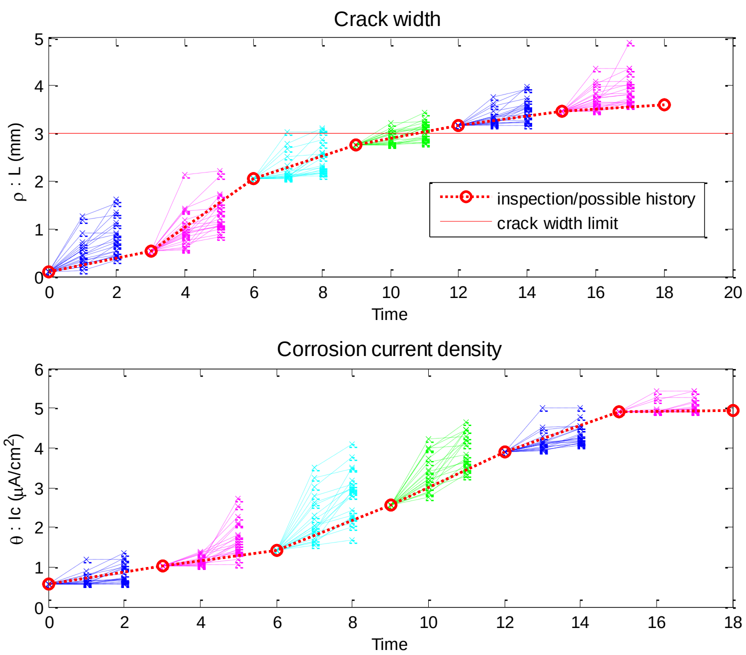
Un des avantages de l'utilisation des processus gamma est que ses paramètres représentent des tendances physiques. En effet, les processus gamma dépendant de l'état permettent de contrôler la taille des incréments de dégradation et par conséquence la vitesse de dégradation. Ainsi, pour modéliser l'effet d'une action de maintenance sur le modèle de dégradation, des paramètres de maintenance ont été intégrés dans les fonctions de forme (§ 4.3.2.1). Cette approche a montré la capacité de modéliser l'effet d'une action de maintenance sur la vitesse de l'évolution de la pathologie (§ 4.3.2.3). En outre, une action de maintenance peut aussi modifier le niveau de dégradation. En effet, le méta-modèle dépendant de l'état profite de la propriété de Markov ce qui permet de donner une valeur du niveau de dégradation après maintenance et ainsi poursuivre la simulation du modèle (§ 4.3.2.2).



Dans cette figure (*cf.*, Figure 4.2), on illustre l'effet de deux types de protections cathodiques (ICS et Galvanique) sur le modèle de dégradation à travers leurs effets sur les simulations moyennes pour la troisième phase de corrosion. Ces actions de protection visent à ralentir la corrosion ainsi que la propagation de la fissure, ce qui est illustré dans cette figure.

Comment prendre la meilleure décision pendant le fonctionnement du système?

Pour répondre à cette question, nous avons pensé aux défis qu'elle évoque. Tout d'abord, dans le temps de fonctionnement du système, les inspections peuvent cibler différents indicateurs physiques : ainsi, des informations sur le système peuvent ne pas être de la même nature à chaque étape du chemin. Deuxièmement, pour prendre une décision basée sur l'état du système, nous devons définir des plages qui limitent des niveaux d'état pour ne pas avoir à gérer une infinité de décisions possibles. Ainsi, pour chaque plage, nous pouvons attribuer une décision. En ce qui concerne la première préoccupation, lorsqu'une inspection manque, une estimation de l'état doit être effectuée. Cette estimation est calculée en utilisant le méta-modèle pour simuler les incréments manquants (§ 4.4). En ce qui concerne la deuxième préoccupation, nous avons proposé une approche pour la discrétisation des plages des états (§ 4.3.3).



Ici, on considère un historique possible d'inspections (cercles rouge) et un intervalle de décision trois fois plus court que l'intervalle d'inspections, soit : $\tau_{inspections} = 3.\tau_{Decision}$.

Dans cette figure (*cf.*, Figure 4.8), on illustre une simulation où on reconstitue les époques non-estimées pour prendre une décision qui sera basée sur la probabilité d'avoir un certain niveau de dégradation.

Conclusions

Nous avons évalué les performances du modèle via des applications à l'Eurocode 2 (§ 3.5.1), à la gestion des risques (§ 3.5.2) et aux politiques de maintenance (§ 4.5). Les résultats décrivant la robustesse et la traçabilité mathématique motivent davantage la recherche sur ce type d'approche en intégrant la prise de décision basée sur le risque et finalement l'optimisation de la maintenance.

Pour conclure sur la modélisation probabiliste de dégradation multiphasique pour l'optimisation de la maintenance d'infrastructures en génie civil, deux avantages pour l'utilisation de méta-modèles ont été soulignés:

- Premièrement, la description du modèle de vieillissement avec une signification physique des principales tendances probabilistes et couplages des entrées (évaluation CND) et des sorties (paramètres de décision). Avec cette approche, nous résolvons une problématique principale : l'absence de relation entre modèles physiques de dégradation de plus en plus compliqués et la complexité croissante de l'évaluation des CND (découplage, fusion ...) avec les développements hétérogènes entre ces deux domaines scientifiques;
- Deuxièmement, dans un contexte CBM, la simplicité de description, flexibilité, calibration et calcul statistique font de ce modèle une approche facile à implémenter et avantageuse à utiliser dans un cadre de gestion des risques. L'évaluation de ces méta-modèles est faite à travers des processus stochastiques dépendants de l'état intégrant des renseignements fournis par CND. L'idée est de faciliter le transfert entre les informations disponibles et le modèle.

Chapter one

Introduction of the thesis

– Keywords –

Probabilistic degradation modelling.

Non-destructive testing.

Condition-Based Maintenance.

Meta-models.

Multiphasic pathologies.

State-dependant stochastic processes.

Effect of maintenance actions.

Decision-making.

1.1 Context and motivation

Humans, for as long as they've existed, seek to make their lifetime longer and expeditious, and their days productive and easier – on average, from a scientific point of view. For centuries they stood against all odds; taught themselves to hunt, planted lands and cultivated crops, contemplated philosophical and spiritual questions, wrote books, constructed bridges and ports to bring the world closer, erected factories to produce for the masses, invented cars and boats and airplanes to travel faster, and now, comes the computer and the internet to bring humans to the closest they've ever experienced to a time where it's almost possible to hear each other's ideas. Along every step of the way, humans encountered numerous difficulties, for example the lack of technological advancements. However, there was a common struggle that was shared between all evolutions, and maybe was the key motive for progress to exist – that is *optimization*.

Civil engineers' essential job is to help humans overcome geographical difficulties; bridges over rivers and valleys, airports, offshore wind fields for green energy, skyscrapers narrowed between towers in city centres to concentrate businesses. They calculate the required materials that a structure needs in order to stand tall for a period of time, knowing that it is subject to dynamic loadings and environmental changings, thus, alteration in behaviours. A structure design's quality depends heavily on the understanding of its future performance; therefore, an essential element of the *optimization struggle* here is to have confidence in the system's future responses to loads, both spatially and temporally.

In civil engineering, structures can be compared to humans; they aim to function for a lifetime and are required to complete a job on a daily basis to serve a higher purpose, *e.g.*, humans progress, and ideally the progress of all living creatures. And like humans, structures require examination and upkeep. Structures' resemblance to humans is growing with the years to a point where nowadays we find structures with complex networks of sensors embedded into them, similar to neural transmitters, whom sole role is to signal deteriorations and faulty joints.

Undoubtedly, civil engineers deal with a vast spectrum of concerns. However, their job continues to grow wider from its basic tasks such as conception, planning, and material optimization – optimal use of quantities of specific materials to attain certain structural requirements. Nowadays, civil engineers face major challenges in terms of upkeep and maintenance.

Maintenance of structures is an increasingly haunting matter for all society, mainly because it is consuming more and more financial and natural resources, resources that are getting limited with time. The increasing number of structures to manage, the growing age of existing infrastructures, and the firmer security and performance requirements also can explain this.

To illustrate the stress, the annual spending on maintenance and repair of national bridges in England is in the order of €180 million, in France the figure is €50 million, in Norway €30 million and in Spain €13 million (BRIME 2001). In Europe, around 40-60% of the construction budget is devoted to repair and maintenance of existing structures with a high proportion of this expenditure on concrete structures, €700 million is spent each year on the maintenance and repair of concrete structures in the UK alone (Stratt 2010). The U.S. Federal Highway Administration (FHWA) released a study in 2002 on the direct costs associated with metallic corrosion. The total annual estimated direct cost of corrosion in the U.S. is $276 billion – approximately 3.1% of the nation's Gross Domestic Product (GDP) (Gerhardus, Brongersl, and Thompson 2002).The ASCE Infrastructure Report Cards of 2013 have estimated a staggering $3.6 trillion needed over a five-year period to improve the United States' infrastructure to an acceptable level (ASCE report card 2013).

Furthermore, given the severe consequences and impacts of a failure, on humans and on the economy (§ 2.1), even on the environment (directly and/or indirectly), maintenance is to be cemented as an integral part of the conception and design of new structures, and to be considered indispensable for old and essential structures. Central bridges, hospitals, airports, ports and dikes are structures that are too important to fail. Monitoring of these structures must be carried out throughout their lifetime, mainly to notify defects and prevent a failure, but also to track the performance and condition.

Figure 1.1 The Øresund Bridge between Denmark and Sweden dips into a tunnel through an artificial island called Peberholm

The European standard for maintenance terminology defines maintenance as the combination of all technical, administrative and managerial actions by which a system is kept in, or restored to, a state in which it can perform its designated functions (EN 13306, 2001). The fact that there are numerous possible combinations of actions – inspection techniques and repairing methods, to mention some – emphasizes the need for optimization of the actions.

For this purpose, we argue that the choice of accurate maintenance actions must be triggered by the system itself rather than periodically. In other words, maintenance is conditional, *i.e.*, based on the condition of the structure. In fact, if every action is carried out at the time of best interest for the structure, within predefined constraints (*e.g.*, performance, cost), it will guarantee better chances at an optimized maintenance policy. This approach is known as a Condition Based Maintenance (CBM), it aims to examine the structure at the right time and with the best technique.

The condition, or safety, of a structure is evaluated in terms of risk associated with the levels of degradation that are generally expressed using degradation indicators. The results of the risk evaluation provide the input for decision-making.

Degradation indicators can be measured using inspections, or estimated using mathematical degradation models. On one hand, the classic approach to modelling degradation is to use physics-based degradation models where physical laws are simulated using computers. On the other hand, non-destructive testing techniques (NDT) offer potentials of the first order but are still not exploited, particularly since they are not taken into account in physics-based degradation models. In fact, connecting NDT to a physics-based degradation model is problematic because the latter has been originally developed for design and conception purposes, and not to integrate NDT measurements. Hence, there is a clear gap between classic degradation models and mathematical probabilistic models that are effective in uncertainty and NDT integration contexts. From this point of view, we see a need for new approaches to model degradation that must surpass the classic approaches, particularly in terms of uncertainty and NDT integration.

To this aim, in this document we analyse conventional damage modelling techniques, especially probabilistic ones, used for maintenance problems in order to point out the required characteristics that define a "good degradation model" in a maintenance optimization context.

As discussed before, maintenance concerns are numerous. The framework of this thesis is to respond to concerns related to degradation modelling. In particular, the general aim is to provide an approach to improve, on a modelling level, the relation between the physical process of degradation, databases and the management of the structure throughout the operation time of the system by taking into account a live feed of information issued from NDT.

In this contextual, we intend to answer the following question:

i. **How to improve the evaluation, modelling and prediction of degradation for maintenance purposes?**
ii. **How to be realistic? What to do with missing inspections and lost information?**
iii. **How to update and model the effect of a maintenance action after a decision?**
iv. **How to make the best decision throughout the operation time of the system?**

One objective of this work is to find an alternative to multi-parametric and multi-physics mechanistic approaches that can be problematic in NDT and uncertainty integration contexts. An increasingly promising replacement is data driven methods, where the degradation model follows the evolution of the degradation, in time or in state, based on data issued from measurements, particularly NDT. These approaches capture the trend of degradation, and can adapt to changes.

An approach to modelling data-driven models that is increasingly being used in this context is through stochastic processes. These processes are incremental in the sense that the increment of degradation follows a defined probability distribution, therefore, they are suitable for accumulative damage commonly found in civil engineering such as wear, corrosion and fatigue. Hereafter, we call this approach "degradation stochastic processes".

However, degradation stochastic processes may encounter numerical difficulties in non-stationary contexts, *i.e.*, when the increment's distribution should not constant in time, in other words, when degradation is not linear in time. However, the speed of degradation varies in time and is affected by the condition of degradation, hence, non-stationarity is modelled using state-dependant distribution laws.

In this thesis, we propose a new formulation of stochastic processes for non-stationary degradation modelling of materials that allows a better integration of measurement data issued from NDT. Also, by depending on the mathematical properties of these stochastic processes, we can improve the speed of calculation in a maintenance context. In other words, we aim to propose new maintenance models for degrading structures based on replacement models, also known as degradation *meta-models*.

We define degradation meta-models as probabilistic models built of a small number of parameters based on the physical expertise and the probabilistic relevance on one hand, and on indicators of degradation and sustainability directly accessible from NDT on the other hand. Meta-model's philosophy is to accelerate the transfer of information between available data and the decision-making by basing the decision platform on a model with a physical meaning which outputs are physical indicators accessible through NDT.

The main objective of this thesis is to show the applicability and promote the use of meta-models or integrated degradation models for decision-making in condition-based maintenance policies applied for structures and infrastructures where data can be accessible through NDT.

One of the main challenges in this study is to join two scientific communities touched by the same concerns of maintenance optimization: On one hand, a community interested in developing probabilistic degradation models, and on the other hand, a community that deals with mathematical problems and optimization.

In this thesis, we focus on cumulative degradations that are generally encountered in civil infrastructures. Rare cases like auto-reparation are not treated. Also, we consider a discrete-in-time framework for decision-making and inspections.

1.2 Outline

In this thesis, we argue a meta-modelling approach for degradation and maintenance. The organisation of this thesis is analogous to what is known in industrial engineering as a “system” approach in which the examination starts with the global system and then spread down to the component level. Hence, we start by analysing the maintenance by identifying its requirements and objectives, then we go down to analyse degradation modelling, that is the backbone of prognosis for maintenance, etc.

In the second chapter – the first being this introduction - we carry out a literature review on maintenance’s essential elements: maintenance management policies (§ 2.2), inspections techniques (§ 2.3) and decision-making approaches (§ 2.3.4). The aim of this first review is to emphasize on the need for new degradation modelling techniques that allow the degradation models to be integrated in new dynamic maintenance contexts, especially in terms of NDT integration, and implementation in complex maintenance systems.

After this first review, in section 2.4 we define the characteristics of a “good” degradation model in a maintenance context. Then, we carry out a literature review to analyse conventional degradation modelling techniques that have been used for maintenance, and we compare their characteristics with the required characteristics.

The literature review of degradation models revealed two major trends: on one hand, we have rich models based on the explanation of the physical degradation, and on the other hand, we have probabilistic models based on statistical quantities (Frangopol et al. 2004). However, the qualities of a degradation model are no longer limited to the ability to model the pathology, but also to the quality of prediction, the ability to integrate new NDT observations, and the ability to be implemented in complex maintenance policies and optimization criteria.

One approach seemed promising for maintenance in civil engineering, that is the construction of degradation meta-models based on stochastic processes such as the gamma process (Van Noortwijk 2009). However, this type of approaches faced serious mathematical identification problems, especially in non-stationary contexts. A proposed solution was to explain this non-stationarity using state-dependant stochastic processes where the evolution of the degradation process depends on the current condition of the structure, and is independent of the time.

In section 2.5, we define the state-dependant approach. To illustrate the approach, we model the case of fatigue crack propagation (*i.e.*, Paris-Erdogan law) using a state-dependant gamma process (SDGP).

Finally, in section 2.6, the second chapter ends with a definition of the degradation meta-model approach that goes broader than just degradation modelling. Within the identification of its elements, we summarize and define the objectives of this thesis.

In the third chapter, the construction of the state-dependent degradation model is discussed within the case of a submerged concrete structure subject to chloride-induced corrosion. Before going through with the mathematical formulation of the model, in section 3.2, we start with an analysis of the said pathology. Corrosion is a three phase pathology: a) diffusion of chloride, b) corrosion initiation, and c) crack propagation. In this first section of the chapter, we look into tendencies and degradation indicators.

A great effort is allocated to the choice of indicators because they are the outputs of the model, hence, the basis of decision-making and risk assessment. The choice of a degradation indicator is based on its weight in the degradation process, and on its accessibility though NDT.

A practical aim of the thesis is to propose a unified approach to degradation modelling that fulfils the characteristics of a “good” model, in terms of maintenance: in section 3.3.1, we propose to model each degradation phase using two indicators, the first indicator should represent a condition level, and the second a potential of evolution. Each indicator is modelled using a state-dependant stochastic process where increments are gamma-distributed on a given time period. Both processes are inter-dependant.

In section 3.3.2, in the aim to pedagogically illustrate the tools for the construction of the degradation meta-model, we apply the unified modelling approach to the third phase of the corrosion process, *i.e.*, crack propagation. The two chosen degradation indicators for this phase are the crack width and corrosion current density, and are modelled using a bivariate degradation model based on two non-stationary state-dependent gamma process.

Data is rarely found in a perfect state in civil engineering. Data is bound to have errors, missing values or lost information. Therefore, in section 3.3.3, two estimations algorithm are proposed: one for complete databases, and the other for databases suffering from missing information.

The rest of the chapter is a set of numerical applications that aims to highlight the properties of the statistical inference process. In section 3.4, we focus on the estimation abilities of the estimation algorithms. While in section 3.5 we propose a study to highlight a benefit of using two indicators instead of one in terms of quality of prediction. Finally, in section 3.5.2, we propose an application of the meta-model in a risk-management framework where we discuss the model as a risk based decision-making tool.

In the fourth chapter, we aim to illustrate the interest of our proposed degradation modelling approach in inspired real world maintenance. Maintenance and management policies are usually focused on minimizing the life-cycle cost only, thus, the optimal solution in this context does not necessarily result in a satisfactory long-term structural performance. Therefore, in this chapter, we use the meta-model to estimate a life-cycle cost and a performance indicator for the structure to pave the road for future work on meta-model updating and maintenance optimization by considering multi-objective optimization policies.

But first, the degradation model must cover the three phases of the pathology in order to be considered in a maintenance policy, since maintenance actions and inspection methods are specific to each of the three phases. In section 4.2, we extend the degradation model to cover the three phases of the pathology, using the unified approach defined and illustrated in Chapter 3.

In order for the meta-model to be considered in a maintenance management system, maintenance elements must be analysed. In section 4.2.3, possible maintenance actions are catalogued, the effect of a maintenance action is modelled and explained, and finally, a performance indicator is defined. Furthermore, in section 4.4, a discussion on decision-making approaches is carried out.

All the previous discussions and analyses allowed us to implement and illustrate the meta-model using two maintenance policies in section 4.5: a preventive policy that aims to prevent initiation of corrosion, and a corrective policy that aims to reconstruct after failure. The proposed maintenance management model aims to estimate a life-cycle cost and a performance index.

In the fifth chapter, the conclusions of the thesis and proposed future works are summarized.

NB – A summary of the thesis in the French language can be found after chapter five.

1.3 Acknowledgment

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Figure 1.2 Région Pays de la Loire's Logo

Chapter two

Probabilistic Degradation Modelling and Inspection in Maintenance

– Abstract –

Degradation modelling in civil engineering is not for the sole purpose of conception of structures, but also to help in maintenance decision-making. Investigations targeting maintenance-aimed degradation models are recent and extensive; however, they remain less developed than conception-aimed degradation models. The use of conception-aimed models in a maintenance management context faces many practical drawbacks such as long running time and on extremely hard calibration process from limited available information. In this chapter, we review the different components of a maintenance management system such as inspections and decision. In this scope, we define and discuss the main characteristics that a maintenance-aimed degradation model should have to be a "good" choice for both modelling degradation and facilitating its integration in a more complex maintenance scheme.

2.1 Introduction

Success and progress of society are secured by the ability to communicate, produce and exchange goods and knowledge. This ability is made possible through what we call infrastructures such as roads, power lines, ports, dams, bridges, and wind farms.

Most of existing infrastructures suffer degradation that in case of lack of conservation can have severe consequences, both economically and on human lives. Maintenance is therefore a very important duty with economic and safety challenges. For example, thirteen people died in the collapse of the I-35W Mississippi River Bridge, built in 1967 and collapsed in 2007. Another example is the case of the Storstrøm Bridge in Denmark built in 1937 where in November 2011 all train traffic was cancelled immediately after an inspection found a poor structure element. The main plan was to repair everything, but there are doubts on the ability of the bridge to handle the future freight traffic. The cost of a new bridge would be more than 3 billion DKK (400 M€). In August 2012 the Danish government allocated funds for the construction of a new bridge (Boéro et al. 2012).

In Europe, a large number of infrastructures were built after the Second World War; consequently it is very common to find structures requiring repairs and rehabilitation. Some repaired structures exhibit poor repair performance and need to be assessed and given a reliable safety level. A third of the steel structures in the Atlantic area were built more than 100 years ago (Duratinet (Boéro et al. 2009)).

Furthermore, the search for a substitute source of energy, motivated research to develop new means of production as well as new locations for this production. One of the most targeted sites in maritime countries like France is clearly coastal areas. Currently, we are witnessing more research focusing on the exploitation of wharves and offshore wind farms. The latter are located in a very aggressive environment for materials and for inspections to be carried out easily. In addition, the investment cost of these means of production makes the costs of exploitation of this energy not profitable compared to the cost of nuclear energy, for example.

All these factors helped stress on the importance of maintenance in civil engineering. Maintenance evolved from being reserved solely for rehabilitating deteriorated structures, to being qualified as an integral element of the design of infrastructures and needs to be specified in advance.

A maintenance management system can be divided into two parts; a) A degradation model to approximate and predict the actual process of ageing in condition or in reliability, and b) A decision model uses the degradation model to determine the optimal times of inspection and maintenance.

To be able to operate in a dynamic maintenance environment, degradation models need upgrades to have certain qualities, *e.g.*, the ability to capture the evolution of the degradation through its reliability performance, the ability to take account data issued from NDT, and to be capable of easily being integrated in a complex decision and maintenance optimization criteria.

We make use of complex multi-parametric degradation models that were developed for representing the main trends, but not for (i) uncertainty propagation, and (ii) updating from NDT results that are generally not directly linked to the model.

Physic-based degradation models are intensively studied in civil engineering but their constant increase in complexity make them harder to use in a maintenance context, especially when the degradation model is asked to be updated with new inspection data. On the other hand, Markovian cumulative damage approaches such as Gamma processes seem promising, but they suffer from lack of acceptability by the civil engineering community due to poor physics-based considerations.

Furthermore, the uncertainties are inherent in every aspect of civil engineering and needs to be taken into account in maintenance management system. Uncertainties are found in material properties, costs, influenced-environment factors (behaviour and impacts on the structure) (Bastidas-Arteaga and Schoefs 2012), imperfect inspections (Sheils et al. 2010), manpower and risk expressed with the degradation level (O'Connor and Kenshel 2013).

We can consider that we have a great richness in degradation models, in technologies of measuring and inspecting structures, in maintenance procedures as well as in decision criteria. However, the combination of all these elements and their integration into the same scheme remains the hardest and a crucial task for any maintenance optimization process. It is within this framework that our literature review lies.

A very helpful technical guide on maintenance related techniques can be found on http://durati.lnec.pt. Also, we can emphasize on the difficulty encountered in some cases between measuring the degradation and the degradation mechanisms. In some cases the degradation is not visible or not measurable using NDT, *e.g.*, the case of degradation motivated by fatigue (roads, *etc.*). Fatigue is an internal process; therefore, we find difficulty in measuring the degradation.

2.2 Maintenance management in civil engineering

Maintenance can be defined in a general manner as "the combination of all technical actions and related administrative correspondences intended to keep an element or restore it to a state in which it can perform its required function" (Besnard 2013). One of the fundamental points of maintenance optimization is the definition of intervention strategies, that must be optimized and based on well-established criteria. The reader is invited to read (H. Wang 2002) or (Castanier 2012) for a detailed review of different maintenance policies.

Maintenance can target different functionality such as reliability, where the structure likelihood to fulfil its functions is to be kept high, durability, where the structure needs to function for a certain period of time, availability or serviceability, where the structure needs to be suitable to use for a certain percentage of the time. Moreover, there is also an economic criteria where a cost-optimized maintenance is also aimed (Van Der Toorn 1996). As a result, to select the appropriate maintenance policy, the decision maker must have a clear set of objectives and constraints. Maintenance needs to be planned and should not be triggered by catastrophic events.

There is a widespread range of planned maintenance policies, however, they can all be classified into two categories depending on their response to failure (or its prediction): Corrective Maintenance and Preventive Maintenance. Corrective maintenance is carried out after the detection of failure and intends to restore the system to a state in which it can perform its required function. Preventive maintenance is performed at predetermined intervals or criteria and aims to reduce the probability of failure and increase the lifetime of a system in an optimal manner. This latter should rely on rational and robust decision aid tools as the decision is based on condition, which is still acceptable for users.

In Figure 2.1, a representation of the two types of maintenance approaches is illustrated. However, this remains a simplified representation since the failure is considered to be instantly detected, which might not always be the case. Inspection intervals have a big effect on maintenance management, therefore, they themselves requires optimization also. A late inspection failing to capture the failure before happening can have severe consequence in Preventive Maintenance.

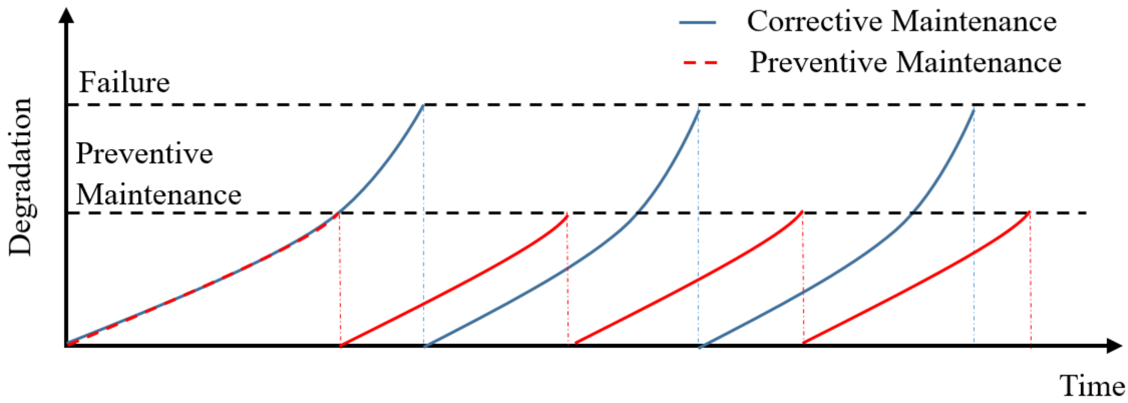

Figure 2.1 Preventive Maintenance Vs Corrective Maintenance

Corrective maintenance is generally used in two cases; 1) where there is no cost-effective way for detecting the degradation evolution of the structure (low consequences for instance), and 2) where components have a defined life span and its failure is predicted. Inspections are becoming more and more developed, therefore, case 1) is not valid for many structures anymore. In case 2) usually the consequence of a failure is not severe and the decision makers are generally aware of this.

For large-scale structures, where the consequences of a failure are severe, it is better to maintain the structure at an optimal level of performance before failure occurs.

An alternative to corrective maintenance is preventive maintenance. Preventive maintenance aims to lessen the likelihood of the failing of structure, or even to keep the degradation from exceeding a threshold. In example, chloride extraction from concrete subject to chloride induced corrosion after corrosion have already started; if the chloride is extracted after the corrosion have already started, that means that sediments of rust are already settling inside the concrete, and are not treated with this maintenance action. If late maintenance occurs few times, the accumulating of rust might generate internal stress. The worst-case scenario is where the accumulation of rust is not taken into consideration.

Preventive maintenance can be carried out periodically or according to other criteria. The two most common preventive maintenance policies are: Time-Based Maintenance (TBM) and Condition-Based Maintenance (CBM) (Mann, Saxena, and Knapp 1995).

2.2.1 Time-Based versus Condition-Based Maintenance

TBM is a preventive maintenance carried out at predetermined, usually periodic (H. Wang 2002), intervals of time and without investigation into the condition of the system. TBM is suitable for failures that are related to the age of the structures where a distribution of the probability of failure can be established, nonetheless, requiring a significant large number of failure observations. Conversely, in a dynamic environment, the use of TBM is controversial since unrealistic assumptions are made where the operating conditions are assumed to remain constant (environmental effects, service, *etc.*). However, this approach can be acceptable when we consider that the environment's variations are not measurable/observable. Furthermore, for discussion purposes, it is possible do define a unique TBM policy respective to structure that is subject to a distinct environment without searching to model the effect but only focus on a global approach of the failure.

On the other hand, CBM is a preventive maintenance that investigates into the condition of the system. CBM is based on the information gathered by the surveillance and the inspection of the system and it consists of three main steps (Jardine, Lin, and Banjevic 2006): the acquisition of degradation data, the processing of these data and the decision of maintenance.

In Figure 2.2, the general process of the two maintenance policies and their position in the planned maintenance policies is presented.

Even though TBM is simpler to implement, CBM is gaining popularity mainly for two reasons; a) its proactive approach, and b) the development of new technologies that can provide a large amount of data. A recent and extensive review on CBM and TBM in maintenance decision-making can be found in (Ahmad and Kamaruddin 2012) where the authors reveal that the application of CBM is more beneficial and practical compared to TBM.

However, CBM requires further research and improvements, especially on the need for a practical and friendly application for the policy. CBM remains more theoretical than TBM, its application in real case scenarios is faced with many limitation, *e.g.*, where we lack of information and data about the evolution of degradation, or where we dispose of information yet we lack in exploiting them, especially when exploiting the relation measure-failure. Another difficulty is encountered when defining a failure or threshold level of degradation.

Nowadays, in maintenance management of vital structures, the use of CBM is strongly recommended if not required, therefore, in the next section we focus on this type of policies.

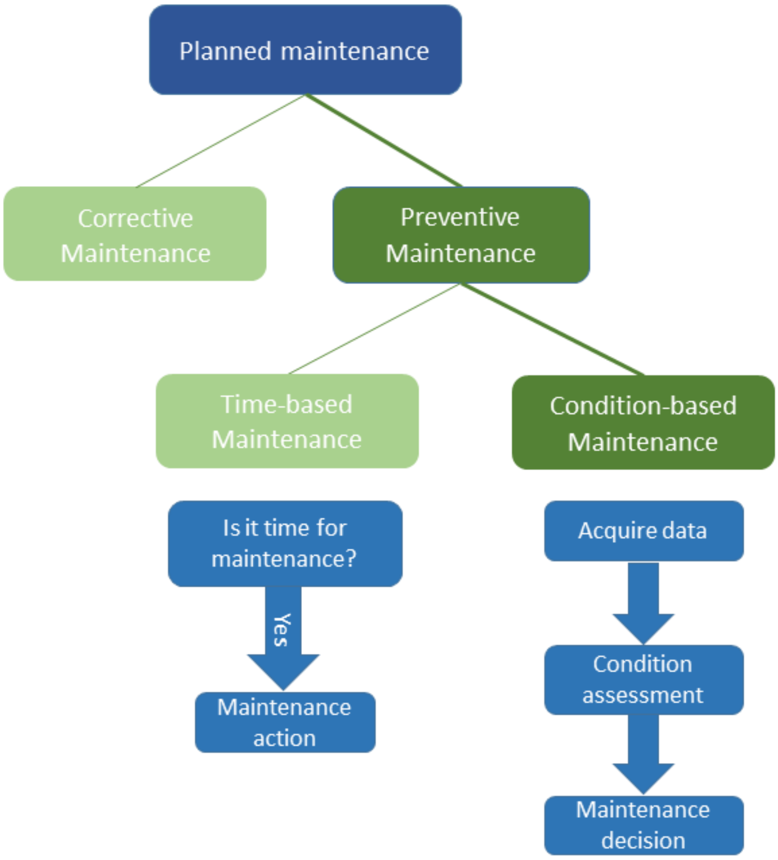

Figure 2.2 Planned maintenance policies, and general process of TBM and CBM

2.2.2 Condition-Based Maintenance

CBM has witnessed a lot of development in the last two decades making it the most popular and intensively discussed maintenance policy in the literature (Farrar and Lieven 2007; Kothamasu, Huang, and Verduin 2009; Sikorska, Hodkiewicz, and Ma 2011; Ahmad and Kamaruddin 2012).

We distinguish between two types of CBMs depending on the frequency of incoming information, in other words the time intervals of condition assessment: a) condition is monitored continuously through sensors, and b) condition is assessed through discrete time inspections.

From a theoretical point of view, it is preferable to detect failure as soon as it happens, prompting the appropriate maintenance action in the optimal time. However, in real life applications, both from an economical and technical point of view, continuous monitoring faces many challenges. Disadvantages and advantages of continuous monitoring are discussed in section 2.3.2 “Structural Health Monitoring”.

The backbone of decision-making in a CBM is failure prediction/estimation based on the current condition, which will be compared to a predefined limit-condition threshold called "failure" threshold in order to prompt the appropriate maintenance actions. Therefore, decisions are made in two manners based on two different types of condition assessment: a) current condition evaluation-based (CCEB), and b) future condition prediction-based (FCPB). The operational nature of these two methods are diagnosis and predictive diagnosis, respectively, both of them aim to prevent a failure from happening. In literature, there is a dispute about the relation between these two methods, and their definition. It is common to find the following definitions: Diagnosis is the process of finding the source of a fault (Jeong, Leon, and Villalobos 2007), while predictive diagnosis or prognosis is the process of estimating/predicting when a failure may occur (Farrar and Lieven 2007). The differentiation between the two is not well defined, especially when prognosis is related to, and highly reliant upon, diagnostics. However, in management system and civil engineering, we use the delineation of Sikorska *et al.*: "diagnostics involves identifying and quantifying the damage that has occurred (and is thus retrospective in nature), while prognostics is concerned with trying to predict the damage that is yet to occur" (Sikorska, Hodkiewicz, and Ma 2011). Diagnostic provides useful information, on which prognostics relies, therefore, the two cannot be completed in isolation, and we speak of predictive diagnostics. However, data requirements for diagnostics are often different from data requirements for prognostic modelling. The professionals who are responsible to decide what data needs to be captured and stored do not always appreciate this.

In the CCEB method, the current condition of the degradation is evaluated, and if it reaches or exceeds a defined limit, the appropriate maintenance action is carried out. Otherwise, the structure is assumed to be in good condition and can still perform its functionality. In most cases, this method is performed via periodic monitoring or inspection in order to collect current condition data.

On the other hand, in the FCPB method, the condition is predicted using the degradation model, and, as in the previous method, if it reaches or exceeds the predefined failure limit, the appropriate maintenance activities are planned and scheduled. Otherwise, the equipment is assumed to be in good condition.

However, Ahmad *et al.* discuss practical limitations to the use of these two methods (Ahmad and Kamaruddin 2012). For the CCEB, if the updated assessment of degradation founds that it has already reached or exceeded the failure limit, there may not be enough time to plan maintenance. This is because this method evaluates current conditions only when new inspections are carried out. Therefore, the decision process suggested via the CCEB method fails to adopt the PM concept, where maintenance is planned before failure occurs. Applying the FCPB method can solve the obvious solution to this limitation; however, the reliability of future predictions remains debatable where the reliability of short-term predictions is higher than that of long-term ones. In other words, FCPB is only useful and reliable for short-term predictions where the degradation is one or two inspections ahead of the prediction.

Prognosis is a difficult task requiring precise, adaptive and intuitive models to predict future condition states (Kothamasu, Huang, and Verduin 2009). Great effort has been made for the development of predictive reliable degradation models. Issues in the growth and maintenance of prognostic systems include the selection of data acquisition and modelling technologies, with considerations including available types of information and approaches to achieve and maintain accuracy of the models and knowledge bases.

Maintenance optimization in CBM

Maintenance optimization is defined as every action done in the attempt to find the optimal maintenance times and actions within a set of constraints (*e.g.*, cost, performance). In this definition, we find three aspects: what maintenance actions and inspections are available, what are the optimization criteria and what is optimal?

Generally, maintenance optimization aims to optimize one of the following criteria:

- Minimize the life-cycle cost;
- Maximize the performance of the structure;
- Minimize the life-cycle cost while maintaining a certain performance level, or
- Maximize the performance of the structure when the requirements for the life-cycle cost are satisfied.

However, most existing maintenance systems focus on life-cycle cost minimization only (H. Wang 2002). Therefore, the obtained solution does not necessarily result in satisfactory long-term structural performance (Frangopol and Liu 2007).

Moreover, the integration of knowledge of the pathology in the model tends to make maintenance optimization even more complex. This complexity will be spread in the optimization criterion, which must, for its part, reflect richness in the associated decision-making context. It is clear that the optimal maintenance decision is in relation with the studied pathology, but also must be taken in accordance with the optimization objectives (minimization of the costs, risks, availability of maintenance means, *etc.*).

2.3 Inspection in civil engineering

The increasing degradation of important structures raises the awareness to a reliable and effective tool for assessment of the level of degradation. In civil engineering, inspections techniques are extensively developed and evaluated, every year more advanced techniques are introduced, and old ones are updated and validated.

Inspections can be carried out following a predetermined timetable or after a faulty visual observation. The choice of a particular technique of inspection is based on different criteria such as the nature of the parameter to be inspected, accessibility to the structure, cost (direct and indirect), after-inspection effect on the structure, and structural code requirements of the country.

We dispose of an extensive range of inspection techniques, of different complexity and forms, recommended by different companies and countries. However, they can be classed into two categories:

i. Non-Destructive Testing (NDT) is a type of inspection that can be used in-situ without altering the structure after it is carried out (§2.3.1).
ii. Destructive Testing (DT) is a type of inspection used for testing of samples in a laboratory to obtain material properties and other detailed information on the condition of the structure and respective degradation processes.

The main difference between a DT and a NDT is their effect on the structure after being carried out. A DT inspection harms the structure by isolating a specimen to test, contrary to a NDT. It's up to the inspection expert to determine whether a DT will affect the performance of the structure or display a bad esthetical effect (not less important in some cases). From a structural point of view, it is preferable to use NDT over DT, however from a managerial point of view an NDT can be costly and in some cases less reliable than a DT especially due to environmental effects (temperature, humidity, magnetic fields …) on the measure itself. The difference between NDT and DT is discussed later on within the context of maintenance (§ 2.3.1.1).

In most cases, inspections are first carried out visually, *e.g.*, for bridges we inspect at least once every two years (Parrillo and Roberts 2008). This type of testing is also known as Routine Inspections (RI). RI are time-stamped photographs or geometric measurements, cheap and quasi-instantaneous to carry out, making them easy to carry out on a regular basis. However, using visual inspections, damages can only be identified when the deterioration becomes visible and in some cases this late detection may generate bigger consequences. Therefore, some degradation processes go unnoticed, on the local and/or global scale. To capture these degradations, more detailed or specific methods of inspection are required to uncover the factual evolution of the degradation, *e.g.*, where corrosion has started in the reinforcement steel inside the concrete and is not yet visible as a crack on the surface. For these cases more detailed inspections can be carried out. These inspections can be DT or NDT, and are usually discrete in time.

In other cases, discrete in time inspections cannot be sufficient to capture the underlying degradation process. Therefore, the next solution and highest level of inspection is where the structures or element is continuously monitored mechanically, chemically and /or electromagnetically. This type of testing aims to better understand the cause, effect and the level of degradation of the structure, and is usually aided by the constructed archives from the previous inspections.

2.3.1 Non Destructive Testing

"A general definition of non-destructive testing (NDT) is an examination, test, or evaluation performed on any type of test object without changing or altering that object in any way, in order to determine the absence or presence of conditions or discontinuities that may have an effect on the usefulness or serviceability of that object" (Hellier 2001).

The most common NDT method is visual testing and usually based on this observation further inspections methods or actions are decided (Roelfstra et al. 2004). A development of this relies on image processing (O'Byrne et al. 2013b; O'Byrne et al. 2014a). We can cite also simple NDT methods that are regularly used such a hammer to detect delamination, or Schmidt-hammer to detect concrete strength, and cover meter to detect the concrete cover (Naumann and Haardt 2003). Simple NDT methods are not enough to uncover inner degradations, therefore more detailed NDT are required for this purpose. Numerous NDT methods have been developed and with different complexity and sophistication.

According to (www.ndt.net) the nine most common NDT methods in order are: Ultrasonic Testing (UT), Radiographic Testing (RT), Electromagnetic Testing (ET) in which they mention the Eddy Current Testing (ECT) and Acoustic Emission (AE or AET). More modern NDT methods are also available, such as Shearography and Active Thermography (Hung et al. 2009) and in many cases a combination of techniques is used for a better assessment (Winkelmans and Wevers 2002). A whole overview of NDT in civil engineering is available in the duratiNet technical platform (www.duratinet.org).

The use of NDT methods is rapidly increasing in civil engineering structures, *e.g.*, concrete and masonry structures (McCann and Forde 2001), wind turbines (Drewry and Georgiou 2007), Ground Penetrating Radar is very common for concrete structures (Parrillo and Roberts 2008; Van der Wielen 2014), Infrared Thermography was applied on concrete and masonry bridges (Clark, McCann, and Forde 2003), for corrosion inspection (Song and Saraswathy 2007; C. Andrade and C. Alonso 1996), different combinations of NDT methods were used for corrosion monitoring (Winkelmans and Wevers 2002).

In the light of the increasing interest and development of NDT methods, they are more and more becoming a necessity and a requirement in civil engineering testing. However, in a maintenance context, NDT suffer from criticism such as being not reliable and that their measurements are always subject to different interpretations, especially because of the lack of normalized protocols: that is one of the goals of the French project DéCoFRé in France (2014-2017). As a consequence, researchers have seen the necessity to lay further control on the manner to carry out and present an NDT, and to this aim industrialized countries in the last 10-15 years have developed training programs to prepare qualified inspectors and also issued codes of conduct of NDT methods. A global review of the qualification and certification of personnel for NDT and condition monitoring can be found in (Thompson 2006).

2.3.1.1 Destructive Testing Vs Non Destructive Testing

The less advertised testing methods are Destructive Testing. DT methods are generally mechanical tests where a specific characteristic of the material is evaluated by isolating a specimen of the structure in a controlled environment. In these conditions, DT measurements are very accurate. However, this accuracy comes with a price. First of all, it is difficult to say that the measurement found in the specimen can be extended to the whole structure: scale effect and representativeness. Second, the tested specimen is destroyed and cannot be neither tested a second time nor put back to the structure, making the follow up in time of the evolution of the measured characteristic impossible. All these limitations for DT are not existing for NDT. In a maintenance and inspection context, both NDT and DT have obvious benefits, however NDT remains more interesting in the long term. For these reasons, NDT is of interest to maintenance optimization, and the earlier literature review has focused on NDT rather than DT.

Hellier has concluded in his book that each testing technique is capable of providing extremely useful information, and to use both of them jointly can be very valuable to the manager. In Table 2.1 we copy an interesting review of the benefits and limitation found in (Hellier 2001).

Table 2.1 Destructive Testing Vs Non Destructive Testing as seen in (Hellier 2001)

Destructive Testing DT		Non Destructive Testing	
Benefits	Limitation	Benefits	Limitation
Reliable and accurate data from the test specimen	Data applies only to the specimen being examined	The part is not altered and can be used after testing	It is usually quite operator dependent
Extremely useful data for design purposes	Most destructive test specimens cannot be used once the test is complete	Every item of the material can be examined with no adverse consequences	Some methods do not provide permanent records of the examination
Data achieved through DT usually quantitative	Require large, expensive equipment and a laboratory	Materials can be examined internal and externally	Orientation of discontinuities must be considered
Various service conditions are capable of being measured		Parts can be examined while in service	Evaluation of some test results are subject to dispute
Information can be used to establish standards		Portable and can be taken to the object to be examined	can be expensive *i.e.* radiography
		NDT is cost effective, overall	Defined procedures that have been qualified are essential

2.3.1.2 Risk Based Inspection

Another application of NDT is in Risk Based Inspection (RBI). RBI is a modern process implemented in Structural Health Monitoring (§ 2.3.2) and maintenance policies aiming to better examine the safety of the structure in order to identify optimal inspection and maintenance strategies (Straub and Faber 2005). RBI are recently developed approaches that emerged and gained popularity in the last two decades and are generally included in a Condition Based Maintenance policy (Arunraj and Maiti 2007; Sheils et al. 2010) (§ 2.2.2), and also used to improve existing maintenance policies (Krishnasamy, Khan, and Haddara 2005).

RBI can be defined as the relationship between the system's degradation model and the available information for estimating the condition of the structure, on which a decision to inspect or not will be based (Rouhan and Schoefs 2003). RBI, and risk based approaches in general, aim to ensure a cost effective strategy that minimizes hazards both to humans and to the environment. Such approaches use information obtained from the study of failure modes and their consequences. Consequences of failure need to be considered by characterizing and evaluating the failure of an event. Risk assessment is therefore an important step of RBI.

Risk based approaches integrate probabilities of failure and the consequence of one occurring at various stages of life time. In this context, risk can be defined as the following:

$$\text{Risk} = \text{ probability of failure } \times \text{consequence of the failure} \tag{2.1}$$

Ideally an RBI tells us where, when, how and what to inspect. This is interesting because it focuses the inspections on elements where the safety, economic and environmental risks are the highest. Also, it identifies the level of inspection or the best inspection technique to be carried out on this element.

It is clear that a RBI is a sophisticated system incorporating many dynamic factors; such as degradation model, catalogue of inspections, risk analysis and assessment. Therefore, in order for an RBI to effectively work, robust degradation models are needed to take into account all inspections, and be able to adapt into this complex framework.

2.3.2 Structural Health Monitoring

Generally speaking, Structural Health Monitoring (SHM) is motivated by the idea that some structures are too important to fail. We have a responsibility towards crucial structures such as major bridges, hospitals, fire stations, power stations and wind farms where a high level of performance needs to be guaranteed. To this aim, rapid and continuous assessment of the state of these structures is important in order to detect damage when it happens and be able to identify its location, size, type and severity.

SHM is a quasi-automated method for tracking the health of a structure commonly applied by installing sensors or measuring devices (*e,g,*. GPS), buried (Shams and Ali 2007; Lynch and Loh 2006) or not, installed in an optimized positions and number (Meo and Zumpano 2005), monitoring the whole structure (global) or a specific measurement of a crucial element (local), transmitting measurements in a live feed to a centre of treatment where received data is analysed and the structure's condition is constantly being assessed.

SHM has been extensively studied in the literature and applied to different fields of engineering, the two most common techniques are: Vibration based monitoring (Carden and Fanning 2004), Guided-wave Structural Health Monitoring (Raghavan and Cesnik 2007). Vibration based techniques were originally developed and used on rotating machinery, however, are now increasingly applied to monitoring wind turbine (García Márquez et al. 2012; Takoutsing et al. 2014) and bridges (De Roeck et al. 2000).

The usage of fibre optics is increasingly being used in civil engineering (H.-N. Li, Li, and Song 2004; Majumder et al. 2008; Schulz, Conte, and Udd 2001). One of the largest structures using fiber optics to monitor its health is the Millau Viaduc Bridge in France where 20 KM of fiber optics is used.

The use of GPS has been recently used to monitor high rising structures (Yi, Li, and Gu 2013). The authors discuss the application of GPS to record the displacement of the structure. However, they criticized the quality of the assessment as being a function of many factors, mainly the satellite visibility. Therefore, the authors emphasized on the need of additional sensors and on rigorous calibration for a perfected accuracy to this monitoring technique.

Also worth mentioning, the 1.1 million euros monitoring system named Wind and Structural Health Monitoring System (WASHMS) and is considered one of the most sophisticated monitoring system where 900 sensors such as accelerometers, strain gauges (Dascotte, Strobbe, and Tygesen 2013), displacement transducers, level sensing stations, anemometers, temperature sensors and dynamic weight-in-motion sensors are used by the Hong Kong Highways Department to ensure road user comfort and safety of three cable-supported bridges Kap Shui Mun, Ting Kau, Tsing Ma (Lau et al. 2000).

Monitoring is one of the most studied subjects in civil engineering and the search for robust and accurate health monitoring techniques continues to grow (Chang, Flatau, and Liu 2003). The literature is rich with studies on different types of monitoring techniques applied to different systems. SHM proved to be a very promising system for managing risks of structure, however the construction and implementation of an effective SHM faces some challenges that are discussed below.

First of all, new types of sensors are introduced to the market yearly (Rice et al. 2010; Nagayama et al. 2010), but their accuracy remains questioned and their measurements are still subject to the interpretations of the controller. Sometimes a higher number of sensors is required, not to cover more spatial variability, but to have more certainty on the measured parameter. As a consequence, a high cost is allocated for the material purchase and installation of sensors. Therefore, the number of required sensors and their optimal location to monitor the desired structural response is a primordial task (Nguyen et al. 2013; Schoefs et al. 2015).

Sensors are small devices implemented within the environment of the structure, making them, in a direct or indirect manner, subject to the same extreme condition as the structure. Sensors are generally electrically operated from a battery or an external generator making them fragile to a blackout where the electrical source is dead. Sensors needs to transmit data to a central computer, therefore, if the transmitting is interrupted due to a broken antenna or cable, it can instantaneously affect the assessment of the condition. However, this is not as dangerous as it sounds in the sense that if a large number of sensors fail, live feed on the condition of the structure is lost, nevertheless, we still have older data and data from the rest of working sensors that can help to make a prognosis of the future's condition of the structure, all while running a maintenance process to revive the totality of the system. We get to the conclusion that also sensors require maintenance themselves in order to uphold their intended functionality.

Cost of sensors makes limitation on the number of sensors that can be deployed; a feedback of the monitoring of existing quays leads to an over cost between 0.8% and 1.3% of the total cost. Nevertheless, a large number of sensors (>100) are usually used on structures, transmitting measurements 24 hours a day seven days a week. As a consequence the broadcasted information is huge and the recordings require a considerably large data centres with high capacities, enough to hold all incoming data in a fast and secure manner. We talk of high data collection costs (sensors, training, *etc.*) (Kothamasu, Huang, and Verduin 2009).

Moreover, we can reflect on the relationship between the duration of life of a structure and the evolution of knowledge on the degradation, in the sense that, with time we find out that the measured parameter is less interesting compared to another one in terms of better understanding the evolution of degradation. In continuous monitoring a structure is generally pre-equipped with sensors for measuring a particular parameter, therefore, the switch to monitoring another parameter can be problematic and complicated, unlike for discrete-in-time monitoring where the switch to a different inspection technique is much easier.

Engineers seek an optimal maintenance policy that satisfies pre-defined criteria in the long term such as performance and costs. Keeping in mind that most structures in civil engineering are slow degrading structures, can continuous monitoring approaches lead to better maintenance policies then discrete time based monitoring? This point remains a heavily debated as long as the cost between the two approaches is as wide as it is today, and is not the focus of this thesis. Jardine *et al.* (Jardine, Lin, and Banjevic 2006) highlighted two main limitations for the use of continuous monitoring; first, inaccurate observation might be acquired caused by increased noise created by the continuous flow of data, second, it is expensive because it requires many special devices. In contrast, the main limitation of discrete-in-time (periodic or not) monitoring is that there's a possibility of missing some important events between monitoring intervals.

2.3.3 Irregularities in databases in civil engineering

In the design phase of new civil engineering structures, knowledge from older and similar projects can be of aid in the better planning of the service life of the new structure. However, such knowledge is not always made available for many reasons such as lack of cooperation between designers and communication between countries. And if it was available, data may suffer from irregularities, values may be missing for many reasons such as incorrect measurements, equipment error, censoring, and manual inputting. Furthermore, a database can be constructed from a combination of databases coming from heterogeneous but similar structures.

Barnard *et al.* (Barnard and Meng 1999) talked about three types of problems that are usually associated with missing data:

i. Loss of efficiency;
ii. Complications in handling and analysing the data;
iii. Bias resulting from differences between missing and complete data.

Loss of efficiency (*i.e.*, the quality of the condition's prognostics) is caused by the time-consuming processes that deals with missing values, and by the fact that the calibration of degradation models require a certain size for a database and a certain continuity.

The second problem, deals with complications in handling and analysing the data. Most estimation algorithms or data treatment methods are not constructed to deal with missing values where databases are out of the pattern. So in a prior phase, they need to be updated and corrected in order to be able to take account for missing values in databases.

The last problem of using missing data in the analysis is the resulting bias from having differences between missing and complete data. When missing data are treated, the values in the new database are not exactly the same as the true values. The new database should not be treated as if it is complete and all its values are true.

2.3.3.1 Missing data

In civil engineering, rarely we can get a perfect set of databases where we don't have missing values. Little *et al.* (Little and Rubin 1987) categorized missing data by the causing mechanism leading to missing values, and it goes as follows:

- Missing completely at random (MCAR), when the distribution of an example having a missing value for an attribute does not depend on either the observed data or the missing data. For example, in a database that includes all inspections, some values are missed randomly from a databases and this does not depend on the level of degradation or the lifetime of the structure;
- Missing at random (MAR), when the distribution of an example having a missing value for an attribute depends on the observed data, but does not depend on the missing data. For example, data are missing from the database after the failure time of the structure (data is ignored), therefore the missing values are predictable. Incomplete databases only due to structural reasons are MAR;
- Not missing at random (NMAR), when the distribution of an example having a missing value for an attribute depends on the missing values. For instance, the inspection testing technique doesn't record values outside a threshold interval. Here we talk about censoring and truncation.

Farhangfar *et al.* (Farhangfar, Kurgan, and Dy 2008) summarize the three major approaches to deal with irregularities and especially missing data. The simplest way of dealing with missing values is to discard the samples that contain the missing values. This approach is only applicable when the number of missing data is relatively small and where the use of the full examples will not lead to bias. A different approach is to convert the missing values into a new value, but this simplistic method leads to serious inference problems. And last, the most common way of dealing with this problem is to impute (fill-in) the missing values. Imputation methods are numerous and specific usually for a certain type or missing data. In their paper, Farhangfar *et al.* (Farhangfar, Kurgan, and Dy 2008) studied the effect of missing data imputation using five single imputation methods. We can also talk of Stochastic Estimation-Maximization (SEM) methods (Celeux and Diebolt 1985), used later on in this thesis (§ 3.3.3.2).

2.3.3.2 Heterogeneous databases

In civil engineering literature, the definition and identification of heterogeneous databases is not always clear. We can think of two reasons why we lack of clear definitions: a) the idea of using heterogeneous databases for the same structural or managerial analysis sounds controversial and inconsistent, so it is automatically abandoned, and b) it's an intuitive procedure taking place in the preparation or conception phase, so heterogeneous databases are automatically used under the premise of being an expert knowledge without thoroughly going through all types of information used in the process.

However, in statistics, heterogeneity arises when describing the properties of a dataset where we have inconsistencies that are believed to originate from several homogenous datasets. Hence, the study of heterogeneity aims at identifying patterns and similarities in datasets and to find sources of disagreement among those results. The study of heterogeneity, or homogeneity, falls into what is known as meta-analysis, or analysis of analyses (Glass 2012), that aims to combine results from different and independent studies.

There is a trend among some researchers that attempt to homogenize the *a priori* heterogeneous databases by seeking to identify covariates (marginal law analysis). This leads to analysing the behaviour of these covariates (per elsewhere stochastic), resulting in more and complex evolution laws. The validity of any distribution assumption remains heavily criticized, and generally for most meta-analysis, it appears that heterogeneity is being consistently underestimated in most meta-analysis (Kontopantelis, Springate, and Reeves 2013). Even though the study of heterogeneity is old (Simpson and Pearson 1904) and extensive (Hunter and Schmidt 2004), Kontopantelis *et al.* have emphasized on the complexity of assessing heterogeneity and the necessity of more developed method (Gavaghan, Moore, and McQuay 2000).

To measure heterogeneity, a classical approach is the use of Cochran's Q test, where the null hypothesis that all studies are evaluating the same effect is examined. Cochran's Q is computed as the weighted sum of squared differences between each study's estimate and the overall estimate across studies, weighting each study's contribution in the same manner as in the meta-analysis. Q is distributed as a chi-square statistic with k (number of studies) minus 1 degrees of freedom (Cochran 1954). The test is known to be poor at detecting true heterogeneity among studies as significant, especially when the number of studies is small.

Cochrane reviews have recently started proposing a new parameter to help readers assess consist easier, the I^2 parameter (Higgins et al. 2003), given by:

$$I^2 = \left(\frac{Q - df}{Q}\right) \times 100\% \qquad 2.2$$

Where Q is the chi-squared statistic and *df* is its degrees of freedom.

This describes the percentage of the variability in effect estimates that is due to heterogeneity rather than sampling error (chance).

A rough guide to interpretation of the I^2 is as follows (Higgins and Green 2011):

- 0% to 40%: might not be important;
- 30% to 60%: may represent moderate heterogeneity;
- 50% to 90%: may represent substantial heterogeneity;
- 75% to 100%: considerable heterogeneity.

In the literature, meta-analysis is mostly used in the medical field and in educational research (Hedges 1982). In Engineering, meta-analysis, in particular the assessment of inconsistencies, is less developed mainly because such studies require bigger databases, nonetheless, we did find some examples for the use of meta-analysis in civil engineering, *e.g.*, slope stability (Travis, Schmeeckle, and Sebert 2011) and construction management (Froese 2009), where meta-analysis are qualified as innovative.

In civil engineering, we can benefit greatly from using information issued from heterogeneous databases in order to better design structure, and in this context define maintenance policies. In fact, some degradation processes maintain a similar course of degradation over their lifetime, for example, in Figure 2.3, we illustrate three commonly seen courses of the degradation; First course (a) is for a slowing degradation process, for example, internal stress from accumulating rust, or crack propagation in a chloride-induced corrosion pathology (Vu, Stewart, and Mullard 2006). The second one (b), an accelerating degradation process, *e.g.*, crack propagation by fatigue (*i.e.*, the Paris-Erdogan law) (Virkler, Hillberry, and Goel 1979). Finally (c), a degradation process that accelerates first then decelerates in the end, for example, corrosion rate in chloride induced corrosion (Yuan, Ji, and Jiang 2009), and concentration of chloride in the concrete.

However, the list of common courses of degradation is not limited to these three, other types may also be found for example shock degradation and linear degradation (Van Der Toorn 1996). Nonetheless, a pathology normally conserves a certain course, for example, corrosion rate from chloride-induced corrosion in reinforce concrete will generally accelerate first, since chloride have full contact with the steel, then after a certain period of time rust will cover the reinforcement steel making the accessibility of oxygen and chloride harder, and the corrosion process starts decelerating (course c).

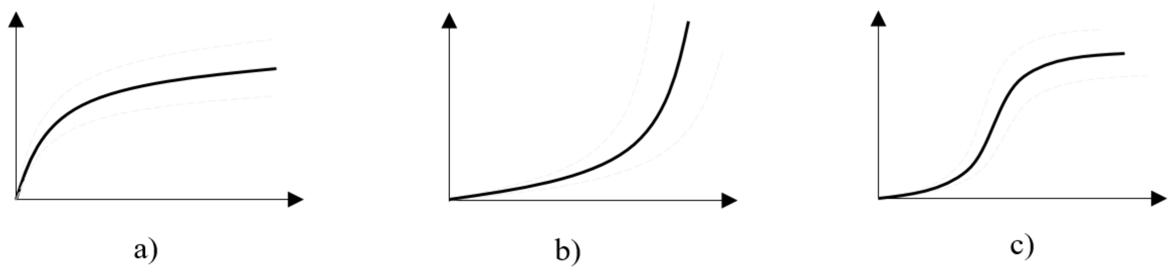

Figure 2.3 Possible degradation courses

The design of many civil engineering components is aided by expert knowledge or expert judgement. With that being said, what is the definition of heterogeneous structures? How do we classify structures according to their homogeneity? And when can heterogeneous structures be considered as expert knowledge and be of aid in construction of maintenance management policies?

First of all, heterogeneity is inevitable in civil engineering. This means that any data originating from different structures are instantaneously diverse. However, let's consider two structures that are constructed using the same characteristics of materials (not necessarily from the same source or supplier), within the same geographical area, where environmental conditions are very close. These structures are very similar. The similarity might help in many ways; pre-require certain inspection techniques, choose a maintenance policy, choose a suitable degradation model, help in the calibration of degradation models, *etc.* We will address this point later in Chapter 3.

We define heterogeneous structures as structures (or components for that matter) that share similar geometry, not necessarily identical, designed to perform the same function in the same manner (a bridge is compared to a bridge), situated in an akin environment (a marine structure cannot be compared to a structure deep on main land), and where loads are close (excluding cases like shocks and accidents).

2.3.4 Decision in civil engineering

An essential task for Civil Engineers is to establish a reliable decision-making process for the maintenance and management of structures in a way that the performance of the structure is maximized while conserving a required safety level and respecting other constraints. Decisions are affected by numerous factors such as economical (minimization of costs, limited budget, *etc.*), probability of failure, consequences, means, social and legal constraints, and last but not least, political. In this section, we limit the study to factors related to unacceptable degradation levels and costs. Nonetheless, all other factors are as important in decision-making; the political factor remains a crucial impediment for engineers and decision-making based on probabilistic assessment, and to convince the real decision maker to believe in probabilistic assessment of safety and risk.

“If all the aspects of a decision problem are known with certainty, the identification of optimal decisions would be straightforward by means of traditional cost-benefit analysis” (Faber et al. 2007). However, in civil engineering, available information is inevitably subject to irregularities and uncertainties from numerous sources such as material properties, deterioration process, and environment. As a consequence, decision-making faces issues in this uncertain context, therefore, probabilistic decision modelling is introduced.

It all started with Von Neumann and Morgenstern (Von Neumann and Morgenstern 1944) where they proposed the theoretical foundations for decision-making regarding situations that involve uncertainty and risk. “Von Neumann and Morgenstern construct a utility function that describes the preference ordering of a rational individual, and show that the individual, faced with uncertainty, ranks actions on the basis of the expected utility of their consequences” (Jongejan 2008). The idea that people maximize the expected utility of rewards rather than expected rewards themselves was first introduced by Bernoulli (Bernoulli 1738).

And all the way till now, decision-making and risk assessment have been almost inseparable in civil engineering (Faber and Stewart 2003). Every decision generates risks and accumulates rewards or benefits; it is up to the decision maker to find and take the optimal decision with the lowest risk and generating the highest benefits.

In a general manner, decision problems in an uncertain context that are expressed in terms of probabilities and frequencies may be treated using the Bayesian decision theory (Schlaifer and Raiffa 1961; Berger 2007; Benjamin and Cornell 2014). Bayesian decision theory refers to a decision theory, which is based on Bayesian probability.

A decision-making basis is the sum of all rules of decision that spans over the lifetime of a structure. An intuitive approach to decision-making is to build a decision/event tree, and depending on the state of information at the time of the decision epochs, three different decision analysis types are distinguished (Faber 2003):

i. Prior: Decision analysis is based on given information where we calculate the expected utility of the decision (Figure 2.4).
ii. Posterior: Same as prior analysis, however, is based on additional information coming from the effect of a decision or after an inspection (Figure 2.4).
iii. Pre-posterior: Decision analysis with ‘unknown’ information spanning on more than one step, where activities and decision are pre-scheduled for the future (Figure 2.5).

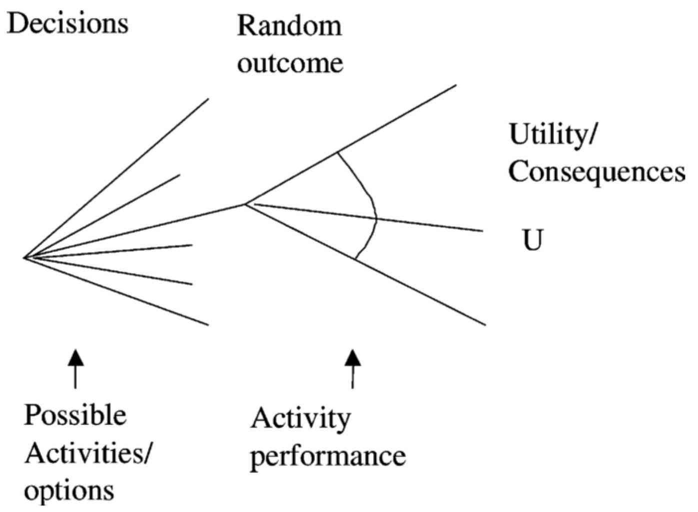

Figure 2.4 Decision tree for prior and posterior decision analysis as seen in (Faber and Stewart 2003)

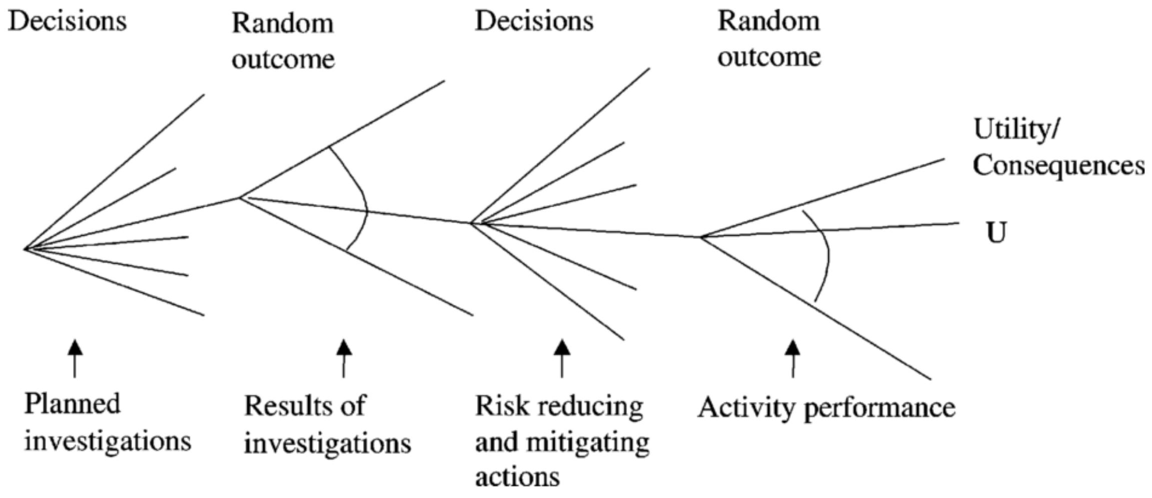

Figure 2.5 Decision tree for pre-posterior decision analysis as seen in (Faber and Stewart 2003)

Pre-posterior analyses form a strong decision support tool and have been intensively used for the purpose of risk-based inspection and decision planning (Faber, Kroon, and Sørensen 1996), however, so far pre-posterior decision analysis has been completely passed over in risk assessments and decision-making (Faber and Stewart 2003).

Decisions, especially in a CBM, are based on the condition of the structure. Therefore, the evolution of degradation in time needs to be tracked in time using reliable and adaptable degradation models, allowing the use of pre-posterior analysis for instance; Degradation models play a big role in decision-making. Degradations are complex phenomena, luckily nowadays we have a better understanding of their evolution in time and better-developed degradation models incorporating more uncertainty and reflecting the physical degradation of the structure.

Once the structure's condition is assessed, the optimal decision must be taken. But, as seen earlier, the decision is not taken without restrictions, the decision maker is generally bound to two types of limitations; 1) predefined-constraints such as costs and a threshold performance level, and 2) unavoidable-errors such as un-reliable assessment of the condition due to imperfect inspections, error in measuring, traffic, and current use. Respectively, decisions aims at two things; 1) to be optimal, that is for a cost-effective and better performing structure, and 2) to be reliable, since assessments suffer from uncertainty.

One of the most commonly used decision models is a Markov decision process (MDP). When the decision for a maintenance action depends only on the current state of the degradation process and not on the time at which the action is performed we talk of Markov Decision Process (MDP) (Kallen 2007). A controlled Markov process describes the dynamics of the MDP. More precisely, a MDP is a discrete time stochastic control process where at each time step t_i the degradation is in some state "s", and the decision maker may choose any action "A" that is available for the state "s". Therefore, in order to be able to outline possible decisions in an optimal policy, a finite set of actions "A", and their details (cost, time, and availability) has to be outlined.

MDP remains a basic and classic approach that has many limitations (White and White 1989). First of all, a key assumption in the definition of the standard MDP is that the decision maker has always access to the value of the current state with no error. However, clearly this cannot be always the case for many reasons. The second limitation involves a state-space explosion, one of the most common problems with MDP models. The third issue with MDP models is the compliance with the Markovian property.

Numerous extensions for the MDP have been developed in the literature through the years, here we mention some of them:

- Partially observed Markov decision processes (Ellis, Jiang, and Corotis 1995; Lovejoy 1991): consider the decision model as an MDP that is based on unobserved states;
- Semi-Markov decision processes (Baykal-Gürsoy and Gürsoy 2007): used for some dynamic systems where time intervals between successive decision epochs is variable;
- Adaptive Markov decision processes (Duff 2002): where there are unknown components that can affect the decision.

For illustration purposes; following the assessment of the degradation of the structure, through diagnostic or prognostic using the degradation model, we have generally the following possible decisions:

1.	Do nothing	No inspections or reparations are required
2.	Inspect further	Assessment is questionable, further inspections are required
3.	Preventive reparations	Prevent or reduce the evolution of degradation
4.	Strengthening reparations	Repair the structure and strengthen if necessary
5.	Corrective reparations	Reconstruction
6.	Decommission of the structure	Abolish the structure

An intrusive decision (#3, 4, and 5), unlike an un-intrusive decision (#1, 2), modifies the evolution of the degradation. As a consequence, the degradation model is altered and needs to take into account this alteration. This comes to emphasize on the need for robust degradation models that can take into account new decisions and integrate their effects on the degradation process.

2.4 Degradation models in maintenance

Degradation models are the backbone of all work in maintenance. The literature reviews on inspections (§ 2.3), decisions (§ 2.3.4) and maintenance management (§ 2.2), stressed out on challenges in terms of degradation modelling for risk management and maintenance optimization of structures. Up till now, the missing part of the previous literature review is on degradation models, therefore, in this section we will discuss different approaches in this topic.

In civil engineering, degradation is a very complex process, affected by many, and not always quantified, factors such as mechanical, chemical and electrical processes. In recent years, degradation understanding has increased substantially and we have a better understating of inspection technique, maintenance and decision. As a consequence, degradation models need to evolve to keep up with the technological advancements in these fields as well.

A literature review of degradation models of structures and infrastructure in a reliability and maintenance optimization context shows two major trends (§ 2.4). On one hand, we have rich models based on the explanation of the physical degradation, and on the other hand, we have probabilistic models based on statistical quantities (Frangopol et al. 2004). Out of this division in models, a recent and interesting trend emerged and is seen as an intermediate or hybrid approach between physical models and pure probabilistic models.

Nicolai (Nicolai 2008) classed degradation models into three categories, or as he calls them boxes:

i. Black-box models, based on statistical time to failure (§ 2.4.2);
ii. White-box model, based on the simulation of the physics of measurable deterioration and failure (§ 2.4.3);
iii. Grey-box model, based on a measurable quantity indicating time-dependent deterioration and failure (§ 2.4.4).

The quality of a degradation model in a maintenance optimization context is strongly related to its ability to take into account the dynamic aspects of the degradation process, and so, in the ability to give a reliable prognostic while considering all possible inputs. As cited previously, prognostic is the estimation of three elements; 1) the level of degradation or condition of the structure at a given moment, 2) the future evolution mechanism of the degradation, and 3) the possible environment where it operates. Furthermore, the model must be comprehensible to represent the important dynamics of degradation for maintenance decision.

As a consequence, in maintenance optimization, the quality of a degradation model is no longer limited to its ability to model the pathology that fits best to the available data and knowledge, but also for its quality to predict the future performance of the structure in a dynamic framework even in different environment or stress constraints, its ability to integrate new observations, particularly from Non-Destructive Testing (NDT), as well as its ease of implementation in complex decision optimization criteria. In § 2.4.1 we define the characteristics of a "good" degradation model. Therefore, we first define the new characteristics of a degradation model in a dynamic maintenance context. Then, a literature review of degradation models used in maintenance is carried out.

2.4.1 Characteristic of the new degradation models

The efficiency of a maintenance management system is highly, if not primarily, related to its degradation model. A robust degradation model must offer a good ability to predict the degradation evolution, both spatially and temporally; therefore it is expected from a model of degradation a set of properties to allow its integration in an up-to-date maintenance management system such as a SHM and Condition Based Maintenance.

We define a "good" degradation model as a model having the following characteristics:

- To model the pathology both in terms of main trends and uncertainty around it;
- To fit available data and knowledge, and to integrate new observations, particularly from NDT inspections;
- To predict the future performance of the structure;
- To be easily implemented in a more complex decision optimization criteria.

Now, let us discuss more in details each characteristic separately.

2.4.1.1 Modelling of the pathology

A degradation model must offer a good ability to predict the degradation evolution spatially.

By definition, a degradation model is a mathematical tool used to represent a “real” degradation mechanism in order to use its output data in post-processing tasks. Degradation understanding has increased substantially, and a degradation model must reflect this understanding by respecting the tendencies and motives of the degradation, especially when we have degradation parameters that are not directly observable, such as, speed of degradation, potential of degradation, un-visible degradation, and accumulation of underlying degradation.

Infrastructure suffer from several types and forms of degradations. Most of them are function of numerous variables that are often un-quantified and affected by changing external conditions. Hence, the use of approaches that are sufficiently generic so that they can adapt to a larger number of pathologies of structures, evolving in a dynamic and uncertain context, seems a beneficial idea.

A "classical" mechanistic approach focuses on the identification and analysis of covariates of environment on the evolution of the pathology. This leads to the development of very specific models, both for pathologies and environments, generally in laboratory conditions where some parameters are fixed and their coupled effects not always investigated our quantified. When such models are not or badly identified, or only poorly controlled (this is the large majority of cases), these approaches are not really "robust" in terms of forecast for a given structure.

Furthermore, most degradation phenomena in civil engineering are multiphasic, that is, different phases of degradation that are motivated by different physical actors (electrical, chemical, mechanical, *etc.*), therefore, treated by different scientific community. In a white-boxes context, the approach is generally to focus primarily on the specific degradation processes of each phase, and then analyse the transitions from one model to another. The "communication" between models is rarely obvious.

Nevertheless, many degradation mechanisms tend to share similar tendencies (Figure 2.3). Foremost, degradation mechanisms are generally slow, and often described as accumulative degradation (Pantazopoulou and Papoulia 2001; Castanier, Grall, and Bérenguer 2005; Ching and Leu 2009; Shi et al. 2012; Si et al. 2013). What is more, most degradation mechanism can generally be broken down into two phases, an initiation phase, where degradation is unobservable on the surface (the use of DT or NDT is required to be able to detect it), and a propagation phase during, which the default is observable on the surface.

Moreover, some authors have strongly stated the importance of incorporating physical meanings into statistical models, especially for data driven models, which are often used in Condition Based Maintenance (§ 2.2.2) (Si et al. 2011). A particular effort needs to be done in order to choose physical indicators that best reflect the underlying degradation process. For example in the case of crack propagation due to chloride-induced corrosion, the parameters of importance are the corrosion current density, and the width of the crack (C.-Q. Li, Melchers, and

Zheng 2006). Also, the accessibility to measuring this parameter needs to be questioned, especially through NDT. It is gainful for a degradation model to have a measurable physical parameter as an output (Chapter 4). First, having a physically significant parameter as an output gives meaning to the degradation process, especially for Civil Engineers. Also, the estimation process is easier to be adapted to real databases of real physical information constructed through inspections.

As a conclusion, we argue and put forward a meta-model modelling approach, associated in industrial engineering field to a “system” approach; Start with the system and then spread down to the component level. In this thesis, the study is practically analogous. We start with analysing the 'maintenance process' to identify requirements in terms of quantities of interest for decision-making and one goes further down to analyse the "degradation" through the available information, different phases, indicators, *etc.*

2.4.1.2 Fit the data

A degradation model should be able to be calibrated using all available information and easily integrate new data.

Degradation models parameters are generally estimated from available data. The estimation process aims to estimate the parameters of the model so that its output is more likely that of the available data. For instance, a time-to-failure model requires failure times to be able to estimate its parameters.

The inherent variability in civil engineering and degradation processes emphasize on the necessity that a degradation model should take account of the uncertainties such as material properties, costs, influenced-environment factors (behaviour and impacts on the structure) (Bastidas-Arteaga and Schoefs 2012), imperfect inspections (Sheils et al. 2010), manpower and risk expressed with the degradation level (O’Connor and Kenshel 2013). Furthermore, data may suffer from missing data from many reasons such as censored data, inspection errors, and lack of communication between governments. It may also be noticed that for a same degradation mechanism, we can have information or data of a different degradation phase and from different techniques. As a consequence, the estimation process can be based on a small amount of “uncertain” information. A degradation model must operate in this realistic context.

Many algorithms are combined with degradation models to allow them to face problems such as errors in the database: 1) imputation methods are commonly used (§ 2.3.3.1), 2) approaches using Monte Carlo simulation, *e.g.*, bootstrapping or the SEM algorithm (Stochastic Estimation-Maximization), which aims to fill missing fill missing, censored or truncated data (§3.3.3.2).

NDT methods are increasingly being used, therefore, information from new inspections are a regular task now and the degradation model needs to be able to take into account in an agile manner all new information to update and improve the calibration of the parameters.

The estimation of the parameters is mostly carried out using classical estimation techniques such as: Method of Moments and Maximum Likelihood Estimation (MLE) (Frangopol et al. 2004; Dempster, Laird, and Rubin 2007; Van Noortwijk 2009).

2.4.1.3 Prognostic character

A degradation model should offer a good ability to predict the degradation evolution temporally.

Maintenance decisions in a CBM are based on the evaluation of the state of the structure through observations. In order to be able to optimize policies, and inspection or maintenance plans, the decision process (and therefore degradation model) should integrate prediction. Therefore, the prognostic of the performance in terms of degradation or risk of the structure is crucial for a maintenance management system. When degradation can be expressed as a change of a measurable parameter over time, it gives the ability to observe this degradation over time.

This point seems, as presented here, obvious, yet lastly; it is very challenging because the observables are not directly linked with degradation. Let remind these captured measures should grasp the degradation process and help establish, interpolate and project the evolution of the degradation over time.

Degradation models might struggle in different ways to fulfil this criterion, here we discuss four of them:

i. Most degradation models focus on the evolution of degradation in time, however, some of them, such as random variables, focus only on the time to failure. In a CBM, a degradation model needs to give a prognosis for the whole lifetime until failure;
ii. In order to capture the evolution of degradation in time, some degradation models require a lot of information and knowledge that is hard to find in civil engineering, *e.g.*, Markov chains and Polynomials Chaos;
iii. The non-linear evolution in time of some degradation processes makes it harder to capture the evolution in time of the model. A stationary degradation model cannot capture the evolution of this degradation;
iv. A considerable influence on the evolution of the degradation is caused by un-observable degradations. These degradations are un-accounted for in the majority of degradation models, therefore, in such cases, the prognosis is not adequate since not all actors are taken into consideration in the evaluation.

2.4.1.4 Implementation in maintenance optimization

Degradation models are the cores of a maintenance management system, and the core must connect all the elements of the system

Maintenance management systems are becoming more and more complex because of the integration of new useful modules such as inspected data from NDT, imperfect maintenance actions, and combination of inspection techniques.

Even though repairing and inspection techniques, particularly NDT, are well developed and the mathematical degradation models are highly developed, the ability to use the two of them for one maintenance problem remains extremely hard for both mathematicians and engineers.

Maintenance actions can have different effects on the structure. Some of the actions do not restore the state of the system to as good as new; we talk about imperfect maintenance or partial repair. In these cases, the degradation model is altered, however, the modification is not easily modelled in degradation model. It is important to have the ability to model underlying and invisible damage processes because it is there where the imperfect maintenance action and partial repairs are stored. For example, the cracking of pavements; before the cracking becomes observable on the surface, there are already small un-observable cracks in the bottom layers. After one periodic maintenance action, the surface of the road is reconstructed and looks new, however the underlying mechanism of degradation is altered, and is not the same as before maintenance (Zouch et al. 2012). An analogue example, which will be detailed later (§ 3.2), can be found in the three phases of the chloride-induced corrosion of reinforced concrete where underlying processes are chloride absorption, rust accumulation and corrosion rate.

2.4.2 Modelling degradation using purely frequency models

Purely frequency models, or black boxes approaches, are based on the relationship between time and failure, easily analysed statistically, but do not provide any deep structural insights (Nicolai, Dekker, and Van Noortwijk 2007). These approached are used where we avoid to understand the physics of the pathology (Kobbacy et al. 1997).

Generally for these approaches, the system can only be in two conditions: normal operation or failure. Thus, generally, these models require no knowledge about the physical process of degradation and only information about the time of failure. However, sometimes covariates reflecting external knowledge into the process can be integrated (Khraibani 2010), still, the approach focuses only on the time of failure, with complete disregard to what happens before.

The classical way to model the failure in this case is to consider a lifetime probability distribution which will be fitted to a sample of observed failures. Parametric distributions such as Weibull distributions are generally used. The objective of this part is not to review all of the models, but to underline the main weaknesses of such approaches in a maintenance context.

In terms of limits, we can compare the two following modelling methodologies. The first is to estimate the overall duration of the structure since its commissioning without additional information other than the possible presence of eventual failure collected during inspections. In its generic application, the expression used to calculate the life expectancy is not actualized. Secondly, the time to failure is modelled by random variables distributed using, *e.g.*, Poisson or Weibull distributions, hence, making them unsuitable for a CBM system that requires by definition the condition of the system before failure. Moreover, it is uncommon in civil engineering to find information on the time of failure for the obvious reasons, unlike the rest of the lifetime where we do dispose of richer information on the mechanism and we tend to benefit from it.

2.4.3 Modelling degradation using physics-based models

Physics-based models, or white-box models (Nicolai, Dekker, and Van Noortwijk 2007), are models based on the simulation of the physics of deterioration and failure.

Since the end of the 1990s, physic-based degradation models have grown to become more and more complex, incorporating physic-chemical and mechanical couplings, leading to an explosion in the number of parameters: for instance, it is common nowadays a 10-parameters propagation deterministic model of chloride ions while Type 90s model included 2 (Rakotovao Ravahatra et al. 2015).

Physics-based models require a thorough knowledge of the physical process of the pathology. Normally these models are complex and built by with a large number of parameters. The advantage of these models is that it gives an actual estimate of degradation, but the downside is the estimation and actualization of its parameters and their associated uncertainties. Nonetheless, in cases where inspection and monitoring is hard and disregarded, the use of physics-based is considered a suitable approach, *e.g.*, subsea situations (Vaidya and Rausand 2011).

For example, in the estimation process of the Two Stage Hit and Grow (TSHG) physical model is in the determination of the probabilistic characteristics that requires a large number of simulations. As a consequence, the calculation of probability of the TSHG process and initiation time distributions consumes a lot of time and this limit is retained for most models of this class of degradation model (Nicolai, Dekker, and Van Noortwijk 2007).

Moreover, the use of the physic-based approaches in a reliability context raises several problems. For instance, how “to randomize” models whose parameters are usually correlated random variables having no prior information on them and how to perform sensitivity studies in the absence of these trends? Rakotova *et al.* have shown that the scatter between models is in the same order of magnitude as the one induced by uncertainty propagation (Rakotovao Ravahatra et al. 2015).

Another common approach relies on the numerical solving of coupled physical equations (Bastidas-Arteaga et al. 2011): the computation time is then incompatible with maintenance optimization algorithms and the authors suggested the building of mono variates meta-models using Markov Chains (Bastidas-Arteaga and Schoefs 2012). An alternative for the model’s calibration is to associate the integration of expertise in the models and the multiplication of the associated experiments, which grows in number as a function of the parameter number.

Furthermore, health-monitoring data are being issued from NDT. The gap between the sophistication of physical degradation models and complexity of non-destructive results is a current huge challenge: one of the consequences is that model inputs and NDT outputs are less and less related. Physic-based models and NDT are not conceived for each other. On one hand, NDT produces one parameter as an output, affected by numerous unmeasured also changing parameters. On the other hand, physics-based models are multi-parametric therefore one parameter output is not sufficient for the proper functioning of these models.

For these reasons, physic-based models face many limitations in a maintenance context, especially CBM. However, these models deepen our knowledge in better analysing and understanding pathologies. It also allows understanding the microscopic effects of loads and constraints as well as the spread of damage within a structure. This knowledge also helps in improving reparation models. However, it requires a thorough knowledge of the constitution of the studied structure as well as the environment in which it operates.

2.4.4 Modelling degradation using data-driven approaches

One approach seems promising for maintenance optimization in civil engineering is the use and development of data-driven degradation models. Data-driven approaches allows modelling the evolution of the degradation using observations via NDT while maintaining the most critical aspects of the degradation mechanism in the model for the decision and an ease of integration in a more complex maintenance decision scheme.

In this category, we gather models based on stochastic processes and can be seen as an "intermediary" modelling approach. The "combination" of the two approaches, probabilistic and physical, creates a robust degradation model, which derives directly from the collected and present databases, and therefore gives a sense to the physics of failure as well as the propagation of degradation. Indeed, for complex systems (*e.g.* offshore wind farms: aggressive environment, temperature, *etc.*) dynamic environments induce changes in the physics of failure, an approach using stochastic processes for modelling the failure offers great flexibility with regard to the description of the mechanisms of failure. This flexibility allows a better description of the databases of failure and a better assessment of the survival rate (Singpurwalla 1995).

Besides, generally it is difficult to obtain the failure mechanisms in advance. As seen in § 2.2, it is expensive and requires a lot of effort to capture the physical process of failure by inspections. On the other hand, approaches based on stochastic processes are seeking models directly from the data of current degradation and previously collected databases. These approaches are more attractive and therefore have gained a lot of attention in recent years (Si et al. 2013).

In this scope, we argue that data-driven models and stochastic processes can be a good solution since they can adapt to different degradation mechanisms.

There is an abundance of models that are motivated by but do not fully capture the underlying physical process of deterioration. We can also mention Stochastic filtering models (Myötyri, Pulkkinen, and Simola 2006), shock models (Abdel-Hameed and Proschan 1973; Esary, Marshall, and Proschan 1973), hazard rate processes (Kobbacy et al. 1997; Newby 1994; Singpurwalla 1995).

Markov chains are widely used to model accumulative damage (Welte, Vatn, and Heggest 2006; Besnard and Bertling 2010; Sun et al. 2012). These models are based on the markovian property, that on the idea that the current states are only dependent upon a finite number of previous states. If the current state is only a function of its immediate past state, it is called a one-step Markov chain model. Stochastic degradation modelling was shown to be feasible with a single Markov matrix (Bastidas-Arteaga and Schoefs 2012; O'Connor and Kenshel 2013). The core component of a Markov chain model is the transition matrix containing the probabilities of transition from one state to another, making its identification in real case scenarios difficult since it requires a large number of transitions in order to estimate all its elements, however, in civil engineering, such information are rarely available. Moreover, Markov Chains require a thorough pre-identification of the states that are widely debated, furthermore, complexity appears when one wants to integrate degradation covariates that can be external (environmental conditions) or internal. Nevertheless, sometimes a stationary Markov chain model solution is considered, meaning that the transition matrix remains the same throughout the specified time period (Frangopol and Liu 2007). Also to be noted, in a maintenance context, temporal discretization of the Markov Chain is not necessarily a problem as these times may represent moments of decision that are generally considered fixed.

These approaches suffer from lack of acceptability by the civil engineering community due to several reasons: absence of data for the model's calibration, poor parameter identification, restrictive assumptions (especially when the degradation show non-stationary characteristics), lack in the application guidelines and in some cases lack of physical meaning (Si et al. 2011).

We can add to this long list of models the Lévy processes. The two most studied and applied Lévy processes for degradation processes in a maintenance context are the Brownian motion (also called the Gaussian or Wiener process) (Guo et al. 2013; Nicolai, Dekker, and Van Noortwijk 2007; Si et al. 2013; W. Wang et al. 2011; Whitmore 1995), and the gamma process (Abdel-Hameed 1975; Singpurwalla 1995; Nicolai, Dekker, and Van Noortwijk 2007; Van Noortwijk 2009; Vatn 2012). However, the non-monotonous evolution of the Brownian motion favours the use of the gamma process in civil engineering. Nicolai *et al.* recommend the use of the gamma process for slow and non-decreasing degradation on the use of the Brownian motion (Nicolai, Dekker, and Van Noortwijk 2007). However, some researchers explain the non-monotonicity of the Brownian motion as a measurement error, imperfect action, or even use its variability to combine with other processes, *e.g.*, the perturbed gamma degradation process, a mix between a gamma process and a Brownian motion (Bordes, Paroissin, and Salami 2010; Chimitova and Chetvertakova 2013).

The gamma process (GP) is a natural candidate for monotonous degradation often encountered in civil engineering. Van Noortwijk (Van Noortwijk 2009) gives an overview of the application of GP within maintenance. Within this field these processes are used to describe degradations such as wear, creep and corrosion. The GP is interesting because it gives to the evolution of degradation a controlled tendency, making its use valid in civil engineering where some degradation mechanism can have identified evolution in time. The stationary GP is extensively studied, however, a main limitation of this process is its linear tendency in time. We may further highlight modelling difficulties when the selected pathologies exhibit non-stationary behaviours over time (acceleration or deceleration in the degradation), or when these behaviours are not only time-dependent but also dependent of the current level of degradation, *e.g.*, in the classical Paris-Erdogan law to model crack propagation by fatigue, the cracking acceleration is dependent on the crack size and not on its initiation time (Paris and Erdogan 1963).

A common solution to this problem to the non-stationary GP in particular, and Lévy processes in general, is by means of extensions of the processes. First extension is the use of non-linear time-dependent shape functions where the increments of the process are gamma distributed with time-dependent shape function and an identical scale parameter (Nicolai, Dekker, and Van Noortwijk 2007). A different approach is to integrate covariates into the gamma process in order to incorporate additional knowledge to the model that attempt to account for the non-stationarity (Singpurwalla 1995; Lawless and Crowder 2004; Paroissin and Salami 2009), *i.e.*, to take into account environmental effects and systems heterogeneity (Bordes, Paroissin, and Salami 2010). Another remarkable approach is the use of conditional or state-dependent models (§ 2.5) to model the non-stationary effects based solely on the degradation levels, where the shape function of the gamma process is state-dependent (Vatn 2012). An extension to this last, an advanced approach dictates in the usage of correlated or multivariate processes (Zouch et al. 2012; Mercier and Pham 2012). However, it may be noticed that for these approaches, in the construction of their model, the authors failed to find a robust procedure for the identification of input parameters as well as a lack of application procedure, limitations making them difficult to appropriate and validate in an operating context (Riahi, Bressolette, and Chateauneuf 2010).

Worth mentioning also are Polynomials Chaos (PC) (Ghanem and Spanos 1990) approaches that are gaining reputation in engineering disciplines by playing the role of a replacement model for physical manifestations that are represented by heavy computational models, *e.g.*, Finite Element Models. However, PC requires a large number of inspections in order to calibrate the model's parameters which is hard to find in civil engineering.

2.5 State-dependent degradation modelling

In this thesis, we are interested in multivariate degradation models where non-stationarity is modelled using continuous state-dependent functions. In these models we aim to represent variables that are carefully chosen due to their degradation-representation quality and their accessibility through NDT. We aim by the building of this meta-model to define a unified approach to model accumulative and continuous degradation mechanisms.

First, we take a step back to position the model relatively to other non-stationary models, therefore, in Figure 2.6 we present a flowchart of degradation models with a particular focus, first, on non-stationary mechanisms, and, second, on state-dependent approaches to model them.

The lack of consideration for non-stationarity in degradation models may lead to the construction of conservative optimized (or non-optimal) CBM policies (decisions in CBM are based solely on the observation of a measure at a time t); same decision will be taken for two structures sharing the same degradation level, regardless of the time of service. To avoid being too conservative, the time of service should be integrated. This, clearly, would increase the complexity of the mathematical formulation, also, makes the decision analysis even more difficult. Therefore, first take a step back to analyse non-stationarity modelling.

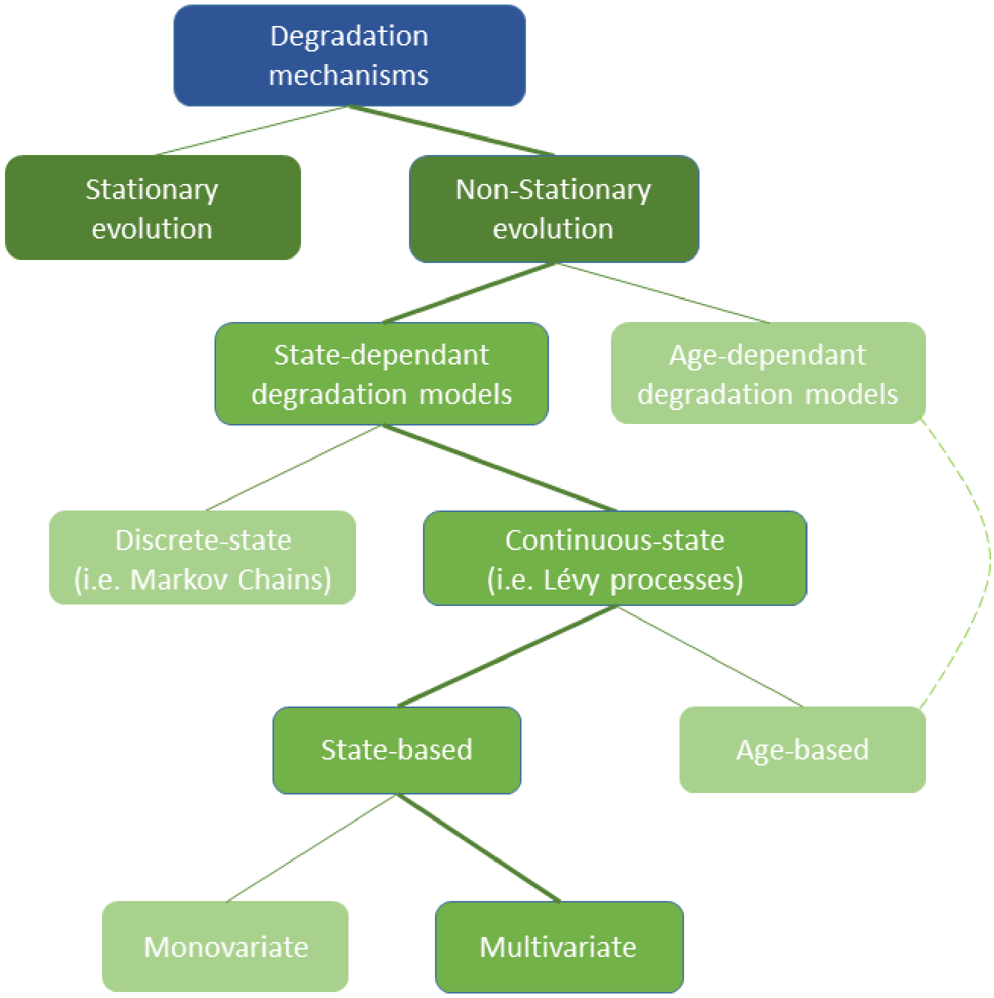

Figure 2.6 Flowchart of degradation models

In section 2.4.4 we discussed different approaches to model non-stationarity, here we are in state-dependent approaches. A classic type of state-dependent models is Markov Chains, models discrete in time and space. In Markov Chains, if we identify the transition matrix, the mathematical problem becomes easily solvable, however, as seen in section 2.4.4, the discretization of states and integration of external covariates into the model is difficult and not easily done.

Therefore, we try to avoid the identification of a state-discrete, discontinuous, probability transition matrix. We will discuss two solutions:

a) A classical approach to model non-stationarity is the use of physical laws such as Paris-Erdogan crack propagation (Paris and Erdogan 1963). These approaches, however, have been criticized by other researchers that they lack to integrate any crack growth mechanisms in them (Wu and Ni 2003). To overcome this, many probabilistic models adopted the crack growth equations proposed by fatigue experimentalists, and randomized the equations by including random factors or processes into them. Here we are interested in random processes in space and time. For example, Lin and Yang introduced the theory of randomization of the Paris-Erdogan law (Lin and Yang 1983), Yang and Manning developed further the model and applied it to cracking data (Yang and Manning 1996; Yang and Manning 1990), Huynh used a similar approach (Huynh 2011), also Zio (Zio 2012). All these last examples lead to a non-linear Markov process with independent, non-stationary degradation increments;

b) Lévy processes, where increments are independent and follow continuous distribution laws such as Gamma, Brownian, and Lognormal. In these models, the increment of degradation on a time interval $(t, t + dt)$, where t is the time since the commissioning of the system, is given by a distribution law of the type $f(t + dt) - f(t)$, where $f(t)$is a non-linear continuous state-dependent function. As said earlier, degradation mechanisms are rarely stationary in time. This trait comes to perplex the choice and construction process of a degradation model. As discussed previously (§ 2.4.4), we dispose of different approaches to quantify and take into account non-stationarity. It has been found that the effects of acceleration may be taken into account using a non-stationary Lévy process, and in the following we will consider a gamma distribution (Van Noortwijk 2009).

The gamma process has been widely used for the description of monotonous degradation processes mainly because of its mathematical properties; for its self-explanatory parameters, α and β, that allows the detailed calibration of the evolution of degradation, and for its monotone and increasing trend in time.

Vatn (Vatn 2012) discussed in his 4 page conference paper a state-based model based on the gamma trend. Zouch (Zouch, Yeung, and Castanier 2011) went forward to propose a bivariate degradation model, however, expressed difficulty in the identification of the parameters of the model (Zouch et al. 2012).

Here, we work in the same framework as Zouch. However, we put a greater effort on the choice and identification of the model's parameter. We propose a bivariate state-dependent degradation model based on the gamma distribution laws, and aims to model a set of thoroughly chosen degradation indicators.

A state-dependent degradation model seeks to translate the stationarity of the process on the state of degradation rather than on time, *e.g.*, the crack growth is function of its size and not the time since its initiation.

Next, we present the main properties of the stationary gamma process, as well as the properties of the univariate state-dependent gamma process (SDGP).

2.5.1 Stationary Gamma process

***Definition 1** – A stochastic process $X = \{X_t : t > 0\}$ is said to be a stationary gamma process with parameters $(\alpha \cdot \tau, \beta)$, where $\alpha > 0$ and $\beta > 0$, if it satisfies the following properties:*

a) $X_0 = 0$
b) X_t *has independent positive increments*
c) X_t *has stationary increment* $\forall\ t > 0$*:* $X_{t+\tau} - X_t \sim Ga(\alpha, \beta)$

Where α is the shape function, β is the scale function and Ga is the gamma distribution:

$$Ga(\Delta X|\alpha \cdot \tau, \beta) = \frac{\beta^{\alpha \cdot \tau}}{\Gamma(\alpha \cdot \tau)} \Delta x^{\alpha \cdot \tau - 1}\, e^{-\beta . \Delta x} \qquad 2.3$$

Where $$\Gamma(\alpha \cdot \tau) = \int_{z=0}^{\infty} z^{\alpha \cdot \tau - 1}\, e^{-z} dz \qquad 2.4$$

The expectation and variance of the gamma process are:

$$E(\Delta X) = \frac{\alpha \cdot \tau}{\beta} \quad \text{and} \quad V(\Delta X) = \frac{\alpha \cdot \tau}{\beta^2} \qquad 2.5$$

And the coefficient of variation is defined by the ratio between standard deviation and mean:

$$CV(\Delta X) = \frac{\sqrt{V(\Delta X)}}{E(\Delta X)} = = \frac{1}{\sqrt{\alpha}} \qquad 2.6$$

The stationary gamma process is a Lévy process, therefore, is infinite-divisible. Meaning that for any integer $n > 1$, there are n independent and identically distributed random variables and their sum has the same distribution. This property is very useful for the optimization of the inter-inspection interval.

As already said before, the linear trend is a limitation to this model, degradation $E(\Delta X)$ is rarely linear in time, but accelerates or decelerates, and this variation in speed is a function of the degradation level of the system. For these reasons the non-stationary SDGP is introduced.

2.5.2 State-dependent gamma process

The state-dependent gamma process (SDGP) is a monotonic non-homogeneous Markov process with independent increments. The increments are gamma distributed with a state-dependent shape function and an identical scale parameter. These properties make the gamma process a suitable candidate to model the non-stationary variability in monotonic phenomena.

The idea behind the construction of a SDGP is to transform the whole non-stationary process to pieces of stationary SDGP. In this case, over a given time interval $\tau > 0$, the increment of degradation follows a gamma distribution function with a state-dependant shape parameter function of the current state of degradation, and τ.

Definition 2 – A *stochastic process* $G = \{G_t : t > 0\}$ *is said to be SDGP with parameters* $\left(\alpha(G_t) \cdot \tau,\ \beta\right)$, *where* $\alpha > 0$ *and* $\beta > 0$, *if it satisfies the following properties:*

a) $G_0 = 0$
b) (G_t) *has independent positive increments*
c) *For a time interval* $\tau > 0$, *we have,* $G_{t+\tau} - G_t \sim Ga\left(\alpha(G_t) \cdot \tau,\ \beta\right)$

However, by releasing the stationarity property of the gamma process, the SDGP loses the infinite-divisibility property and not a Lévy process anymore.

To illustrate, next we discuss the construction of a SDGP for crack growth by fatigue (Paris-Erdogan Law).

2.5.3 Illustration on the Paris-Erdogan law using the SDGP

In this section we illustrate the use of SDGP in the case of crack growth by fatigue. The Paris-Erdogan law (Paris and Erdogan 1963), or Paris law, is the simplest model that governs the propagation of the crack by fatigue or cyclic load, defined as follows:

$$\frac{dx}{dt} = C \times (\Delta K)^m \qquad 2.7$$

Where C and m are material dependent parameters, ΔK is the variation in the intensity of stress on a cycle factor, x is the crack width and $\frac{dx}{dt}$ is the crack width grow rate.

Fatigue crack growth is non-linear in time and therefore cannot be modelled using a stationary gamma process. We profit from this non-stationarity to show a simple example of the applicability of the SDGP. This illustration is done in three steps; first, we create a database by randomizing the Paris law. Then the model is calibrated and fitted to these databases, and finally, the SDGP is simulated. This example is programmed under Matlab©.

2.5.3.1 Generating the database

For the generation of a database, we randomize equation 2.7 and then we extract a part of the data as if they were inspections. For the randomization part, an approach introduced by Lin and Yang (Lin and Yang 1983), and applied in different ways in the literature (Wu and Ni 2003; Yang and Manning 1996; Yang and Manning 1990; Huynh 2011), was used where the authors insert a random factor into the equation, *e.g.*, Log-normal and normal distributions.

First, it can be demonstrated that ΔK can be defined as a function of the crack width and a constant as follows (Myötyri, Pulkkinen, and Simola 2006):

$$\Delta K = \gamma \times \sqrt{x} \qquad 2.8$$

Where γ is a constant (Whittaker and Saunders 1973), that can be estimated through experimental data.

Then, a white noise is introduced in equation 2.7:

$$\frac{dx}{dt} = e^{\omega} C \times (\gamma \times \sqrt{x})^m \qquad 2.9$$

Where ω follows a normal distribution law $N(0,\sigma^2)$ (Zio 2009).

For a very small dt, equation 2.9 can be discretized into an easily simulated Markov-like process.

$$x_i = x_{i-1} + e^{\omega} C \times (\gamma \times \sqrt{x_{i-1}})^m \times dt \qquad 2.10$$

Equation 2.10 is used to “randomize” the Paris law and create the information used to extract data from.

We define a fixed inspection interval τ, where $\tau = 10 \cdot dt$. For each inspection, a point is extracted from the corresponding simulation of the randomised law, resulting in a database with fewer points that could be construed as recorded inspections.

In Figure 2.7, the extracted database and the randomized Paris-law are presented. For this illustration, we use the following parameters: $C = 0.05, m = 1.1, \gamma = 1, \sigma = 0.8$.

After the consolidation of the database, formed of 10 simulations, we proceed to the second part of the program where we define an appropriate SDGP and estimate its parameters.

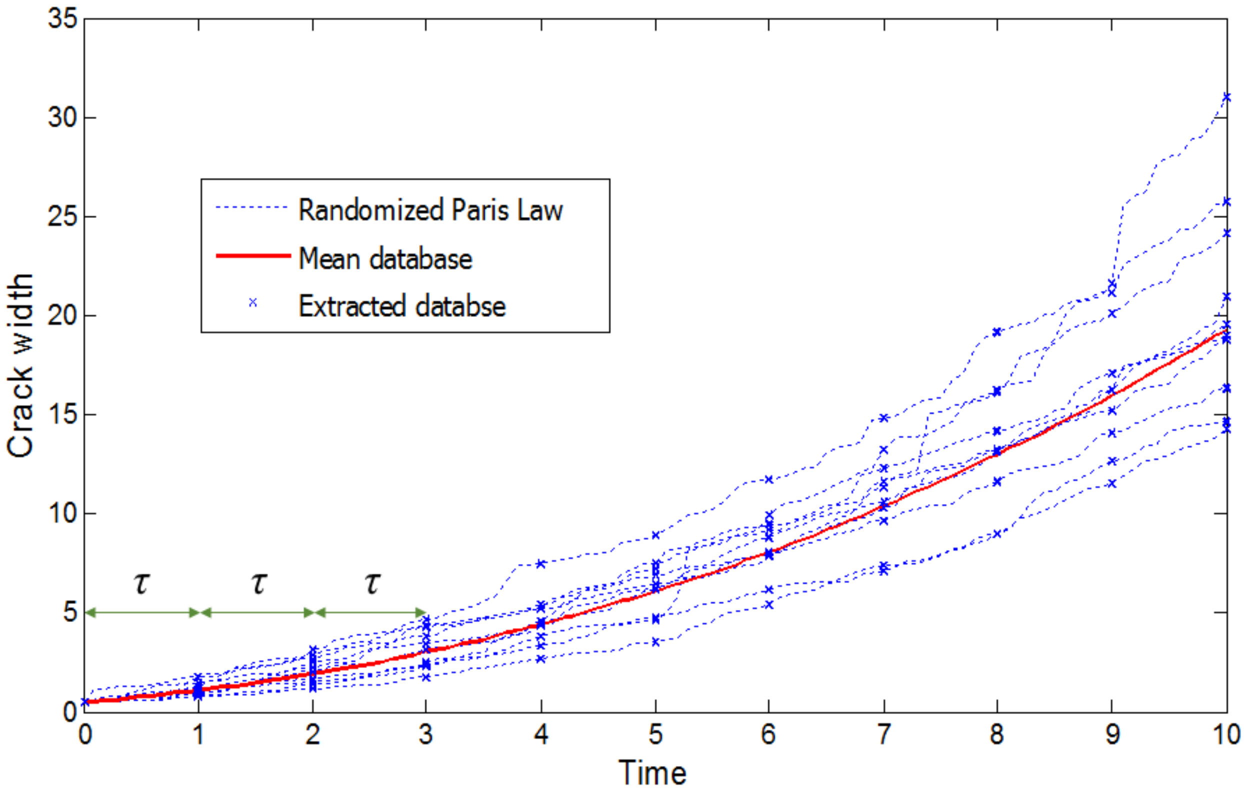

Figure 2.7 Generating the database by extracting values from randomized Paris law simulations

2.5.3.2 Estimation of the SDGP

Our objective is to define the two parameters α and β of the SDGP, and estimate them from the database. The shape function here is considered to be state-dependent and function of the inter-inspection interval τ. To simplify this illustration, we consider the scale function β as constant for all states.

The mathematical formulation of the shape function is based on a choice. α can be constructed as the underlying speed of the process. Therefore, by looking at database in Figure 2.7, the process is constantly accelerating, thus α is required to behave the same. A simple accelerating function that is commonly used in civil engineering is the power law. Hence, and by choice, we define the shape function $\alpha(x)$ as follows:

$$\alpha(x) = (a \times b^x) \times \tau \quad 2.11$$

The parameters a, b and β of the SDGP are then estimated using the classical maximum likelihood estimation algorithm on the database. In this example, we got the following estimates: $a = 5.1968, b = 0.4920, \beta = 0.1569$. In Figure 2.8, the shape function of this illustration is illustrated.

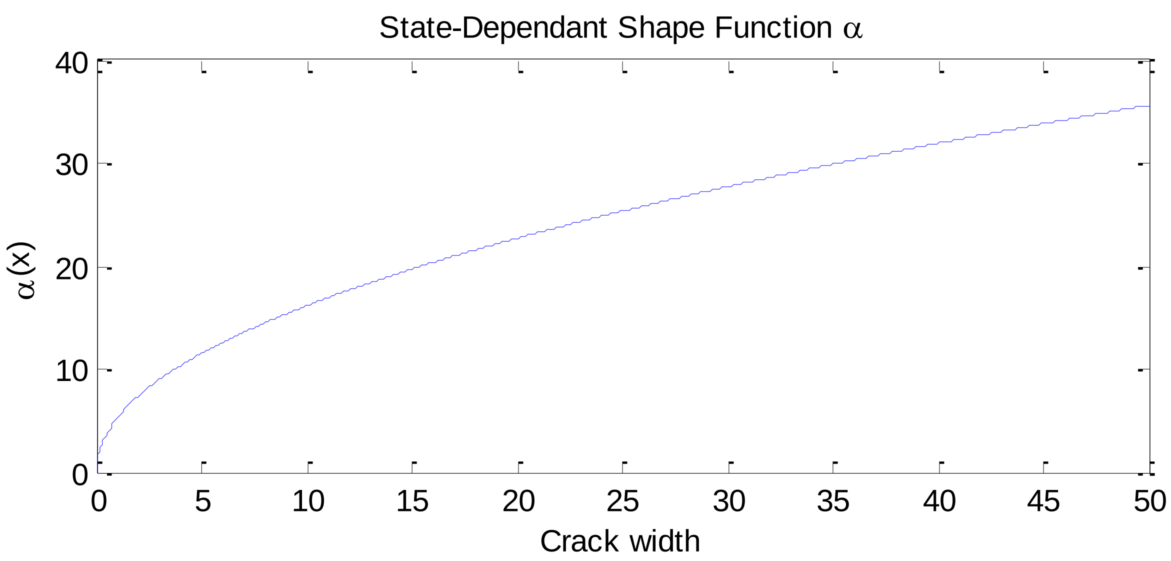

Figure 2.8 Estimated shape function for the Paris-Erdogan crack propagation law

2.5.3.3 Simulation of the SDGP

Finally, we get to the simulation of the SDGP using the estimated parameters. We will expand the simulation time to give a prognostic of the crack growth after the database ends. In Figure 2.9, the database is shown and compared to the simulated process.

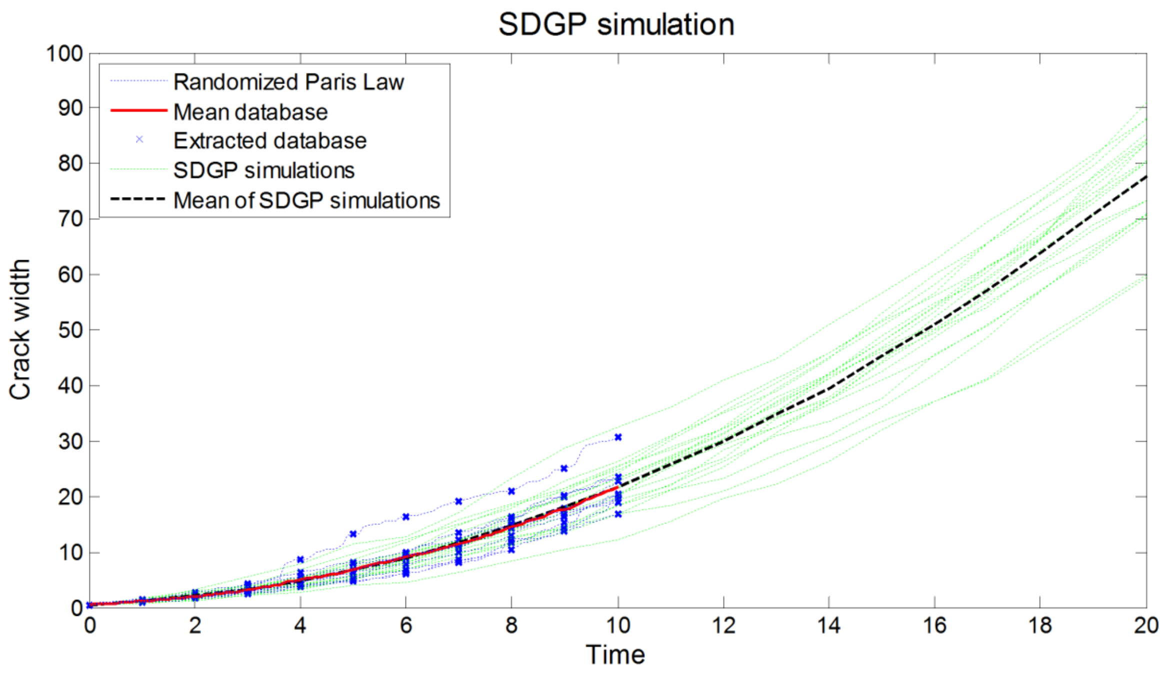

Figure 2.9 Estimation, simulation and prognostic of a SDGP applied to a randomized Paris law

From Figure 2.9, we can see that the simulation captures the underlying database. The two means are perfectly superposed with an $R^2 = 0.9984$. In terms of distribution, the SDGP capture the variability of the database, especially for values that are far from the mean line.

This univariate SDGP demonstrated simplicity in construction and agility in calculation, however, does not meet all the characteristics of a "good" degradation model defined in § 2.4.1. Matter of fact, imperfect maintenance changes on the mechanism of degradation cannot be always modelled using a univariate process (*e.g.*, pavement cracking). A proposed solution is the use of bivariate process (Chapter 3) where we have two degradation indicators that can be modelled, helping in modelling an invisible acceleration or deceleration in the case of an imperfect maintenance.

2.5.4 Why the Gamma Process?

The gamma process, originally called "gamma wear process" (Abdel-Hameed 1975), is reputed for its ability to model accumulative damage that monotonically grows over time, such as corrosion, erosion wear, and creep of materials, which are the most common causes of failure of engineering components (Van Noortwijk 2009).

First, by definition, a state-dependent gamma process (SDGP) is not a gamma process. More correct definition are: a monotone non-homogeneous Markov process with gamma distributed independent state-dependant increments with an identical scale parameter, or a non-stationary gamma process with a state-dependent shape function. However, to simplify here, we call the process "gamma process", and it refers to the general approach to model SDGP.

The use of the gamma process approach is a choice to be made. In this section we discuss what makes the use of the gamma process a good one. Many ideas, from a mathematical point of view, have already been discussed earlier, nonetheless, here we will discuss further within the applicability of the gamma process in the maintenance of structure and infrastructures. Thus, we attempt to answer the recurrent question: **Why the gamma process?**

Benefit #1 – To begin with, the gamma process is based on two explanatory functions that gives a mechanical meaning to the model: the shape function (α) and the scale function (β). First, the shape function, dictates the size of the increments, hence, the evolution of the degradation in time. A shape function can be seen as the speed of variation of the model, *e.g.*, a S-shaped evolution in time requires a bell-shaped shape function ((El Hajj et al. 2014),§ 3.3.2). Second, the scale function, dictates the dispersion of the evolution of degradation. The scale function is generally considered constant for simplicity, however, this hypothesis might be discriminating towards changing dispersions in time. As a consequence to these aspects, the gamma process approach gives a physical significance to the degradation model, plus, it allows the controlling and adjustment of the model to fit pre-known tendencies of pathologies.

Benefit #2 – The gamma process has the Markov property, *i.e.*, the current state is only function of the state before. The main advantage of the Markov property is that it allows the use of the model without having all the history of the structure, but only the current state of degradation.

Benefit #3 – In civil engineering uncertainties taint everything, hence, homogeneity does not exist. An advantage for the use of the gamma process, besides having the inherent stochasticity, it captures the underlying process and adapt to every process by preserving a certain individuality to each dataset. This property is mainly the consequence of the state-dependency in the model, which is reinforced with the bivariate process. This last point is discussed in § 3.4.2, also, a study where we considered heterogeneous databases in the estimation process, illustrates a fast convergence of the model. In other words, since this model is state-dependant, the shape function a “memory” of all increments, as a result, conserves the degradation tendency by slightly changing the parameters to adapt to new heterogeneous degradations.

Benefit #4 – The effect of a maintenance action on the degradation process can be evaluated and quantified directly in the shape function. In fact, the mathematical formulation of the gamma process allows to identify the tendencies of the degradation process (§ 3.3.2). As a consequence, the physical evolution of the process can be manipulated through the shape function using control parameters that can model the effect of a maintenance action (§ 4.3.2).

2.6 Conclusion and Objectives of the thesis: Meta-Modelling

A degradation model should not act as an impediment between available information and decision-making.

Maintenance has become an integrated part of the conception of future structures, and not solely used for already degraded structures. The backbone of any maintenance management system is its degradation model. Therefore, degradation models evolved with the evolution of other elements of the management platform such as inspections, activities and decisions.

A CBM maintenance policy is effective in the sense that it allows the monitoring of the evolution of degradation, and it defines the execution of a maintenance action (inspection and/or reparation) at the best time according to the actual state of the system. One of the main contributors to the efficiency of the CBM model is the degradation model. Without recalling the overall discussion, we assume that, to be the most effective, the degradation model should be a trade-off between the "technical" aspect of the degradation integrating the different challenges previously discussed, and the maintenance decision-making framework. For this, a top-down/bottom-up approach for the construction of the degradation model will be developed. This approach is based on the following definition of the meta-model which finally defines the objectives of this thesis.

We define here a meta-model as a degradation model based on:

- A small number of parameters
- The probabilistic pertinence and physical expertise
- Indicators of degradation and durability directly accessible from NDT

Hence, a meta-model is based on three axes, where each one of the axes needs to be developed in order to get to a robust degradation model that responds to the new defined criteria (§ 2.4.1). The three axes to develop are: degradation analysis, modelling analysis and maintenance modelling analysis. In Figure 2.10, we represent the organisation of analysing and building a meta-model.

First, in ***degradation analysis***, the pathology is studied by looking into its tendencies, significant degradation indicators and expert knowledge. Degradation might be monotone or not, increasing or decreasing, subject to potential maintenance actions or unrepeatable, inspectable through NDT or inaccessible. In this tier of the meta-model, the physical sense of the degradation is studied and introduced into the meta-model. A great effort is allocated to the identification of degradation indicators that can adequately account for the process of the degradation of the structure. Indicators are chosen based on two criteria; a) their accessibility through NDT or DT, b) weight on the degradation evolution.

Second, in view of the degradation analysis, a ***modelling analysis*** is carried out where we discuss suitable degradation models to connect all the elements of maintenance (inspections, maintenance actions and decision), while respecting and reflecting the physical nature of the degradation.

Last but not least step, the ***maintenance analysis*** is carried out where we look into and catalogue different types of inspection and maintenance actions. Moreover, the effect of a maintenance action on the evolution of degradation is discussed.

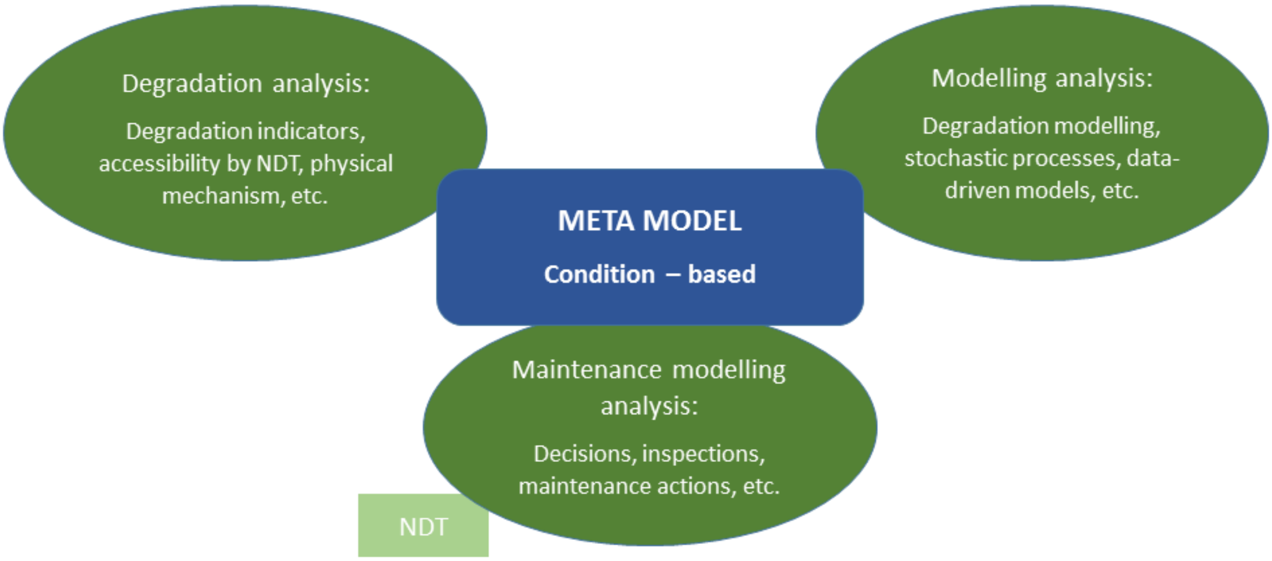

Figure 2.10 Meta-model

The major contribution of this work is to improve upon multi-parametric mechanistic approaches by working on integrating imperfect measures in maintenance optimization strategies based on meta-models. Meta-models aim to use a small number of data-driven "physical" parameters chosen in a way to assess an accurate degradation level on which the maintenance decisions will be based.

In the next chapter, we will apply and discuss the meta-model approach within an application to the cracking of a submerged concrete structure subject to chloride-induced corrosion. The degradation model is based on two correlated SDGP constructed from NDT data. Expert knowledge is introduced to reflect the main useful degradation properties that the model should tackle for decision-making.

Chapter three

Maintenance oriented degradation model

– Abstract –

Physic-based models are intensively studied in mechanical and civil engineering, however, their constant increase in complexity makes it difficult to incorporate NDT models because the two models are treated by two different scientific communities. Moreover, an increase in complexity and number of parameters leads to a huge number of studies for probabilistic modelling of uncertainties. As a consequence, such degradation models are inflexible in a maintenance context, especially where degradation models must be updated from new inspection data. On the other hand, Markovian cumulative damage approaches such as Gamma processes (Van Noortwijk 2009) *seem promising, but they suffer from lack of acceptability by the civil engineering community due to a poor considerations of the physical mechanisms* (Si et al. 2011)*. In this chapter, we introduce the state-dependent degradation model based on the gamma trend, a degradation model that can be seen as an intermediate or hybrid between physical-based models and purely probabilistic models. Furthermore, degradation mechanisms are rarely stationary in time, here, we propose to model this non-stationarity via the state-dependency of the model. The construction of a bivariate state-dependent degradation model will be discussed within an application of the cracking of a submerged concrete structure subject to chloride-induced corrosion.*

3.1 Introduction

The evolution of the requirements for degradation models in a maintenance context have evolved substantially in the last two decades (§ 2.4.1). Complex degradation models are more and more replaced by substitute models that perform better in dynamic frameworks and respond faster to decision-making. In this chapter, we come from this point of view, and we keep the spirit of building a model that aims to *adapt* to available databases and gives valuable information for decision-making as output, all while being updated using information coming from NDT.

As portrayed in the conclusion of the last chapter (§ 2.6), the construction of a condition based meta-model relies on three analyses that need to be carried out in a consistent manner in order for the model to perform properly in a demanding, dynamic maintenance context. Here, in Chapter 3, we will focus only on the first two analyses, degradation analysis (§ 3.2) and modelling analysis (§ 3.3), leaving the last piece, maintenance modelling analysis, to Chapter 4.

Meta-models are often criticized that they lack of detailed processes to help in the identification of the degradation characteristics in order to assure adequacy to the studied mechanism (Zouch 2011). Thus, this study proposes to investigate the construction methodology of the condition-based meta-model, and to test its applicability to degradation problems and to available databases.

This chapter focuses on the construction procedure of the state-dependent degradation model. In section 2 we will introduce and analyse the cracking process of this submerged reinforced concrete structure and look into degradation indicators. In section 3 we will detail the construction of the degradation model based on the different NDT data. The model parameter estimation algorithms are also provided and discussed. In section 4 we conduct a set of numerical analyses to highlight the statistical inference of the model properties especially its estimation abilities. Section 5 is devoted to different applications of the model. And finally, the conclusions of this work are drawn in section 6.

The construction of the state-dependent degradation model will be discussed within the application of cracking propagation of a submerged concrete structure subject to chloride-induced corrosion.

This chapter is based on two articles submitted in this thesis: El Hajj *et al.* (2015a and 2015b).

3.2 Degradation Analysis of Chloride-induced corrosion

Reinforced concrete (RC) is the most widely used construction material in civil engineering. Yet, the understanding of its long-term performance is faced with many difficulties such as the uncertainty impact of the environment on its behaviour, making the assessment of the durability of RC structures in time a complicated task.

A RC structure is subject to physical, mechanical and chemical degradations; Physical degradation results from extreme temperatures, mechanical degradations are mostly due to excessive charges and shocks, and chemical degradation is often the most critical ones since they tend to change the concrete constituents and matrix by reaction (dissolution, swelling). The air, water and soil are known to be agents promoting the migration of contaminants into the concrete, entering into the pores of concrete and thus altering its characteristics, in particular, the chemical composition of the pore solution (Silva 2004).

In this study, we focus on submerged RC as seen on ports and offshore structures. Sea waters are home to many aggressive agents, excluding accidental leaks, chlorides remains the most brutal one of them. Chlorides are responsible for one of the main mechanisms of degradation of existing structures: *corrosion*. Corrosion of reinforcement steel is known to be one of the major causes of degradation of reinforced concrete (RC) structures (Bastidas-Arteaga and Schoefs 2012).

Chloride ingress into RC structures leads to serviceability and safety losses. Degradation modelling allows the estimation of the effects of chloride ingress, with regard to serviceability and limit states. Ultimate limit states are highly dependent on both, geometrical characteristics (cross-sectional dimensions, span length, *etc.*) and loading (dead, live, seismic, *etc.*). Therefore, to generalize the results, this work focuses on a serviceability limit state related to the time to corrosion damage of the concrete cover (severe cracking or spalling).

In RC, the concrete is associated to steel reinforcements, therefore, the concrete, with his high alkalinity, plays the role of a physical barrier and a chemical protector for the steel: a guardian micro film grows on the surface between the concrete and the surface of the steel to protect to steel from corrosion. This film is effective only when the pH of the concrete is high (pH > 13) (Shi et al. 2012). The chlorides react with the concrete chemical components and result in lowering its pH. When the concentration of chlorides in the concrete exceeds a critical threshold (Angst et al. 2009), the corrosion kinetics are active and the result is initiation of corrosion, if not treated, it can result in significant damage (e.g., loss of steel, cracking) that affects the durability and stability of the structure.

Considering the consequence of a failing structure, many models have been developed in the literature to characterize and predict this phenomena. These models usually aim to predict the time when we reach each phase of the degradation and performance of the structures, however, most of them don’t follow the evolution of degradation (through physical indicators for example) in time.

Chloride-induced corrosion process of RC structures can be divided into three phases:

- **The diffusion phase** is characterized by the diffusion of the aggressive agent, here chlorides, in the concrete. When the chloride concentration on the surface of the reinforcement steel exceeds a threshold concentration (Angst et al. 2009) we reach steel depassivation where the protector film is breached, and we have initiation of the corrosion.

- **The corrosion phase** is dominated by the expansion of corrosion products (rust) in which they slowly expand and fill the pores surrounding the reinforcement steel. Rust fills a bigger volume than the denser steel where it originates from. In consequence, after a period of time, rust has no more pores to fill and starts generating internal stress on the concrete until the first hairline cracks appear (0.05 mm width). At this point, the crack deterioration phase is reached.
- Finally, **the deterioration phase** lies in the excessive accumulation of corrosion products. This generates extreme tensile stress and results in crack propagation of concrete cover till rupture. According to Table 7.101N from Eurocode 2, for an exposure class XS3 (Corrosion of the reinforcement induced by chlorides from sea water) the performance of the reinforced concrete structure are assumed modified when the width of a crack is greater than or equal to 3 mm.

The three phases of the chloride induced corrosion are shown in Figure 3.1.

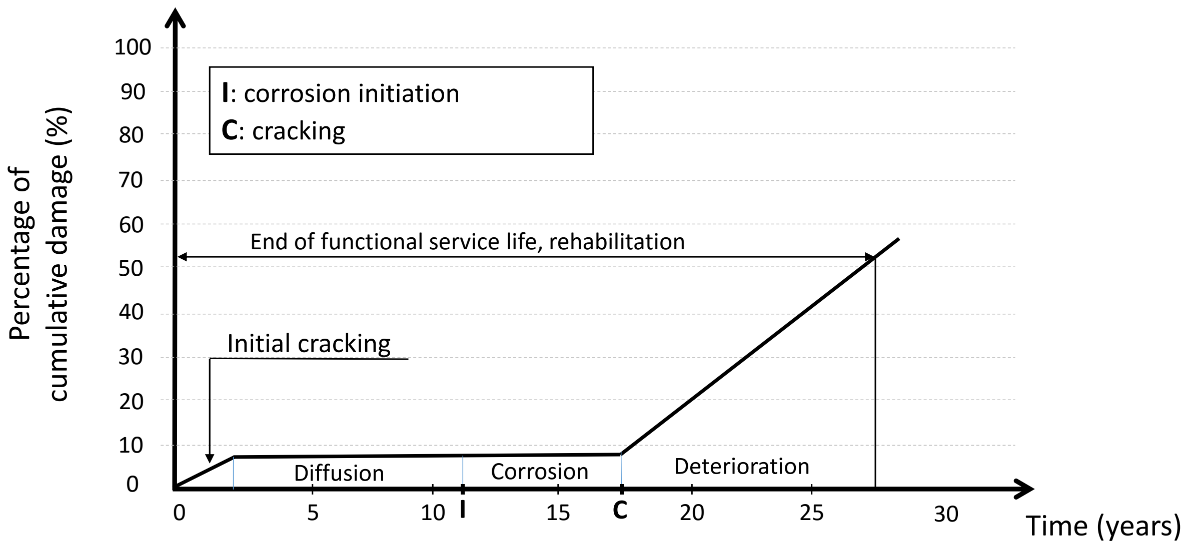

Figure 3.1 Degradation by corrosion of reinforced concrete by (Youping Liu 1996)

NB – Cracking may appear after a fatigue loading too, however, in this study the propagation of cracks is assumed to result from the corrosion solely.

3.2.1 Selection of degradation indicators

The construction of the degradation meta-model is based on a state-dependant processes that represent "physical" indicators of the pathology. Therefore, in the construction of the model, as it will be detailed later in this chapter, the degradation is based on carefully chosen physical indicators that are suitable to be used in a maintenance-aimed degradation model. A great effort is made for the selection of these indicators in a way to represent the best the degradation process. Here, we discuss the choice of the criteria.

Several techniques have been developed to measure and monitor the evolution of degradation in concrete. In the case of a RC structure, the reinforcement steel has indeed strengthened the material property, but can also be a cause of new cracks because of its own corrosion. The aim in choosing suitable indicators is to represent reliably this pathology at every instant in time (or decision epoch), of every phase of the process.

Degradation is a complex process that includes numerous factors, some of them changing with time (*e.g.*, crack width), and some of them are unchanging with time or quasi-static (*e.g.*, concrete cover). All factors have an impact on the degradation, however, only the changing ones can be associated with a method to track the evolution of degradation. And eventually, the unchanging factor's effect is indirectly represented through the rest of the indicators.

Changing indicators are classed into two categories: internal and external. Internal indicators, *e.g.* crack width, represent the actual level of degradation, and, external ones, *e.g.* temperature, represent an external factor that plays a role in the evolution of the degradation. For the degradation model being built here, we are interested in choosing internal indicators for three reasons:

i. Internal indicators give a true level of degradation of the structure if inspected, rather than an estimation of the degradation in the case of external;
ii. External indicators have an impact on internal ones, therefore, they are indirectly included in the model by means of the effect on the degradation;
iii. Decisions are based on the true condition of the structure, *i.e.* an individual degradation level. In fact, external indicators are internal indicators to other process, *e.g.* temperature is an internal indicator of climate change.

For every phase, there are *n* potential indicators to represent it. However, we want to consider only the important parameters in terms of decision and degradation tracking, thus, we need to restrain the choice to the best ones. Here, we propose to choose two indicators per phase. We want the two indicators to be the most adequate to represent a degradation phase; between them, the two indicators must keep information about the degradation level, and the potential of evolution. Hence, we ask the questions, what makes and indicator a good choice?

An indicator's value in a maintenance-aimed degradation model is in its observable character and in the value of the information that this indicator can give us, especially in terms of state and growth of degradation.

Therefore, the choice of an indicator is based on two things:

i. Observable character and accessibility through NDT methods;
ii. Significance or weight of indicator in representing the degradation process.

It is important to point out a limit of this approach. In the proposed selection of indicators, we took into consideration only two parameters out of a potential n parameters to model the degradation process ($n > 2$). As a consequence, some indicators are left out of the model (environmental parameters *e.g.* humidity). From a classic mechanist point of view, this approach may be criticized as it leaves out information issued from the indicators. However, the proposed model is not a physics-based model (no physical laws are modelled). The proposed model can be seen as a data-driven state-dependent Markovian process (it has the Markovian property), consequently, we are propagating both the degradation process and its history. Therefore, the left-out parameters are in fact indirectly included in the process (O'Connor and Kenshel 2013).

3.2.1.1 First phase

The diffusion of chlorides phase is governed by the concentration of chlorides in the concrete, precisely, the concentration at the steel interface (Angst et al. 2009).. Therefore, the first indicator has to be the concentration of chlorides, denoted hereafter $[Cl^-]$.

As it is known from earlier, the main reason for the initiation of corrosion is the depassivation of the protective layer around the steel. This depassivation occurs when the pH in the concrete is lowered below a certain threshold. It is known that the $[Cl^-]$ lowers the pH in the concrete (Hurley and Scully 2006). Therefore, the pH is considered as the second indicator.

The measuring of chloride ingress has been extensively studied for years, and numerous NDT techniques can be found in the literature (Torres-Luque et al. 2014).

The pH measures the acidity vs. basicity. The pH scale ranges from 0 (battery acid) to 14 (liquid drain cleaner). The classic test for pH is by spreading a phenolphthalein solution on the concrete, since phenolphthalein is a pH indicator. Nowadays there are numerous advancements in NDT to measure pH (Räsänen and Penttala 2004), *e.g.* Wagner Meters' pH Meter.

A great debate on whether we fix a threshold for the $[Cl^-]$ or for the pH to mark the end of the first phase. Currently it is agreed that the initiation of corrosion is governed by the chloride concentration at the surface of the steel (Angst et al. 2009). There is no clear relation between the threshold of chlorides and the pH, but is known that the threshold varies with pH (Angst and Vennesland 2009).

To conclude, the $[Cl^-]$ on the steel's surface, is the "important" indicator on which transition to the second phase, and maintenance decisions are based (Angst et al. 2009). On the other hand, the pH brings insight on the evolution of degradation allowing for the decision to be based on both indicators instead of one.

Hence, for the first phase, the $[Cl^-]$ is the indicator of degradation, and the pH is the indicator of the potential of evolution.

3.2.1.2 Second phase

The second phase is governed by corrosion. The corrosion's chemical reaction produces rust residue. Once the rust fills the pores around the steel, it starts generating stress on the concrete. If this stress exceeds the tensile capacity of the concrete, the concrete starts to crack.

This phase's description allows us to discover three potential indicators:

i. Corrosion indicators: Corrosion rate or corrosion current density;
ii. Corrosion products indicators: Rust mass, steel loss;
iii. Mechanical indicators: Internal stress.

First of all, the corrosion indicators are the impulse of this phase. They tend to dictate the rate of rust production. Moreover, corrosion current density (i_{corr}) are measurable via NDT, therefore, i_{corr}, is an indicator for this phase.

The corrosion rate, V_{corr}, represents the volumetric loss of metal per unit of area and unit of time (mm year^{-1}); it can be obtained non-destructively from the corrosion current density, i_{corr} (μA/cm^2) through Faraday's law and the density of the metal. i_{corr} is an instantaneous rate of corrosion measured using a non-destructive technique, highly sensitive to external conditions (temperature, humidity, *etc.*) (Breysse et al. 2009). Thus, it is important to note that for modelling and decision support calibration curves depending on the environmental conditions must be available or that inspections must always be carried out under similar environmental conditions and for a given preceding period (similar recent history). We assume the second condition is verified, the first one has not been established for all situations and it is only available in few study cases (Breysse et al. 2009).

Figure 3.2 represents the variation of the corrosion rate on all three phases of the cracking. According to the corrosion electrochemical principle, the corrosion rate is proportional to the corrosion current density (Yuan, Ji, and Jiang 2009).

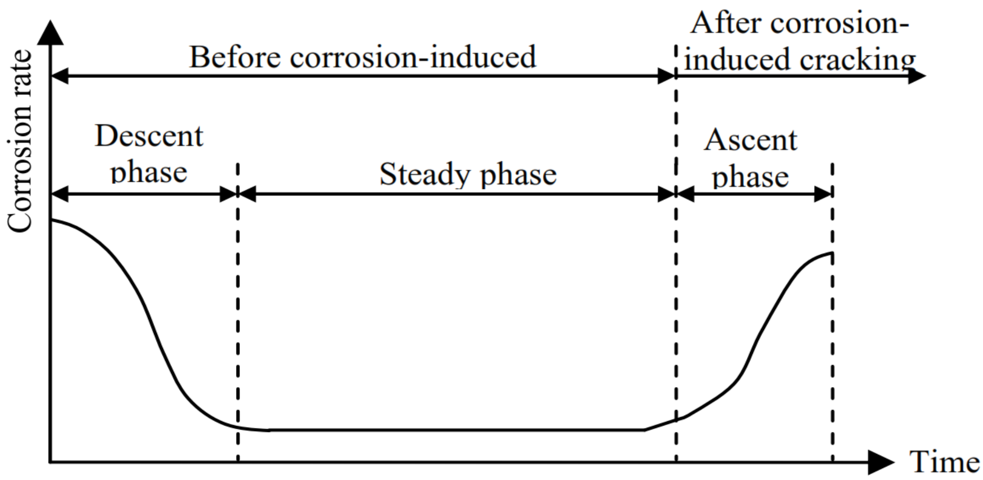

Figure 3.2 Variation of corrosion rate (Yuan, Ji, and Jiang 2009)

Secondly, mechanical indicators, *e.g.* internal stress, allow us to define the end of the second phase in a precise manner. In fact, the internal stress dictates the end of this phase by comparing it to the concrete's ultimate tensile strength. Internal stress measuring is easily carried out via NDT methods using stress gauges implemented in the concrete.

On the other hand, we have the corrosion reaction's products indicators. To measure a product of the reaction we must measure the active elements of the same reaction. Therefore, although rust and steel losses are important acting factors of this phase, to calculate both of them we need to measure i_{corr} first. Transformation of mass is calculated using Faraday's laws of electrolysis (§ 4.5.1.2). Hence, it is enough to consider i_{corr} as an indicator.

Nonetheless, an important advantage of the product indicators is that it records the history of steel loss. This can be very important in cases where we consider imperfect maintenance actions such as concrete replacement of the second level (§ 4.3.1.3); where concrete is replaced after corrosion started but the steel is not changed. In fact, if corrosion has initiated, steel has suffered loss and this loss needs to be registered to keep track in time if the steel diameter is sufficient to perform its required performance. This case is discussed in details in section 4.5.1.2.

Similarly to the last phase, for phase two, the concrete cracks because of the excessive internal stress. The internal stress is the indicator of degradation. The corrosion current density reveals the speed of the filling of rust that causes the tensile stress to increase, hence, is seen as a potential of evolution.

3.2.1.3 Third phase

Parameters of importance during the crack propagation phase are the corrosion current density and the width of the crack (C.-Q. Li, Melchers, and Zheng 2006). Rust generation and steel loss are subject to the same discussion carried out in the last section.

The width of the crack describes the evolution of the propagation, and the corrosion current density is the potential of its evolution. The two indicators are observable and dependent of each other. The crack is considered reachable and its width is easily measured by the mean of gauge block or image analysis: (O'Byrne et al. 2013a; O'Byrne et al. 2014b).

In Figure 3.3, (Vu, Stewart, and Mullard 2006) drew the shape of the variation of the width of a crack versus time for two cases of corrosion rate (time-invariant or time-varying) and two coatings.

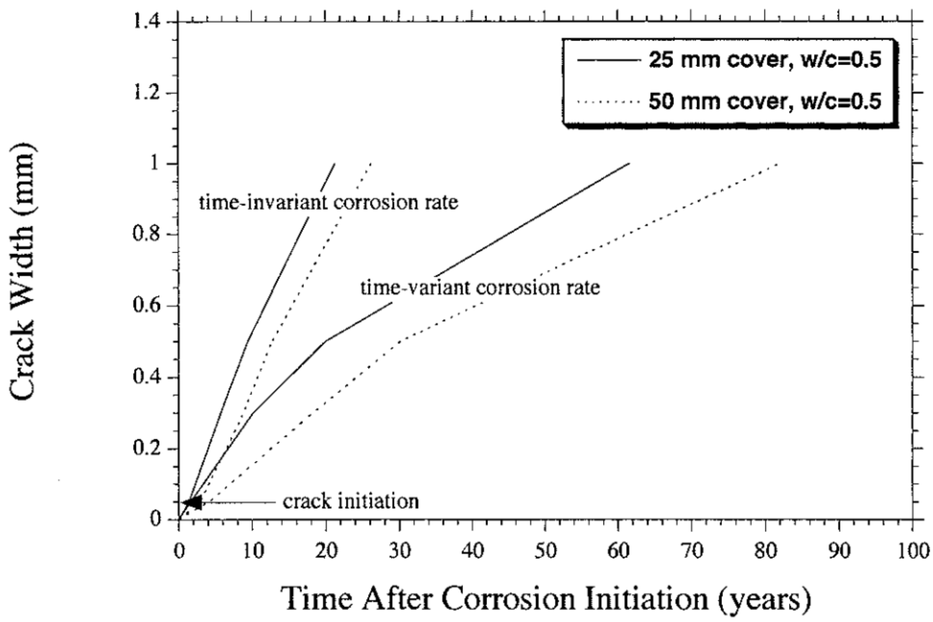

Figure 3.3 Variation of the width of crack - Phase 3 (Vu, Stewart, and Mullard 2006)

The importance of modelling the corrosion rate by a cumulative process is clear. The hypothesis of an invariant corrosion rate is too conservative: in 20 years, it leads to an overestimation of 50% of the crack width.

Finally, for the third phase, we have the crack width, which is the degradation indicator, and defines failure when it exceeds limit crack width, w_{lim}. Then, i_{corr} is named potential of evolution, and is the motive behind the continual accumulation of rust that propagates the cracking.

3.3 Modelling analysis

In the last section, the degradation analysis is carried out where the mechanisms of corrosion were studied and the two degradation indicators of importance (corrosion current density and the crack width) were identified. Now, the next step is ***modelling analysis*** where a degradation model needs to be chosen in accordance to model the degradation tendencies, while respecting the characteristic of a new degradation model.

We want a non-stationary bivariate degradation model that explains the non-stationarity via state-dependent continuous functions; therefore, an appropriate degradation model is the stated-dependent gamma process (SDGP). As a conclusion, the two degradation indicators are models using two interdependent SDGP.

This section is in two parts. In sections 3.3.1 and 3.3.2, we introduce a unified approach to model the three phased phases of the corrosion process, then the construction of the SDGPs in the case of the third phase of chloride-induced corrosion is detailed. In part two (§ 3.3.3), the estimation procedures are presented. Since databases are rarely complete and almost always suffer from some sort of fault, a Stochastic Estimation-Maximization (SEM) imputing algorithm was developed. Nonetheless, for the case of complete databases, a basic estimation heuristic based on the Maximum Likelihood Estimation (MLE) method is first presented. This heuristic was also integrated in the E-step of the SEM algorithm.

3.3.1 Introduction

We propose here to define the degradation for each phase as bivariate process, where each process is a state dependent stochastic processes similar to the approach presented in (Zouch, Yeung, and Castanier 2011). Let the bivariate process be written, written $\left(\rho_{i,t}, \theta_{i,t}\right)_{\forall t\geq 0}$, where:

- $\left(\rho_{i,t}\right)_{\forall t\geq 0}$ describes the condition state
- $\left(\theta_{i,t}\right)_{\forall t\geq 0}$ the potential of its evolution
- i is the number of phase $i \in [1,2,3]$

The two processes $\left(\rho_{i,t}\right)_{\forall t\geq 0}$ and $\left(\theta_{i,t}\right)_{\forall t\geq 0}$, hereafter written ρ_i and θ_i, are both dependent and observable.

The evolution of degradation over a period of time is given by positive increments for the degradation processes respectively $(\Delta\rho_i, \Delta\theta_i)$ which are continuous random variables. We assume that the degradation increments in a given time interval τ are random variables which are a function of the present degradation state (ρ_i, θ_i).

The degradation process is therefore assumed to be a Markov process. As discussed in section 2.4, a suitable candidate for the distribution laws of each increment $(\Delta\rho_i, \Delta\theta_i)$ is the gamma distribution (Van Noortwijk 2009) defined by two parameters *(*α and β where: α is the shape parameter and β is the scale parameter). In the described bivariate state-dependent model, the shape parameter α_i and the scale parameter β_i are independent of time.

To simplify the identification step, we will consider that the state dependency is exclusively governed through the shape functions as function of the current state (ρ_i, θ_i) and τ_i. On the other hand, the scale functions β_{θ_i} and β_{ρ_i} are considered constant throughout the life cycle. By choosing β_i as constant, it is as we considered a constant variability over time. Although both parameters of the gamma process affect the variability of the simulation, β_i remains the one with the biggest impact (Equation 2.5).

Therefore, in order to construct the bivariate model, we have to define and identify the parameters of the shape functions α_{θ_i} and α_{ρ_i} which are respectively proportional to the expected values of the increments for θ_i and ρ_i (§ 2.5.4, benefit #1).

Now, the correlation or dependency of the bivariate model is modelled in terms of mutual acceleration effects directly in each of the shape function of the gamma distributions. This model is sequential in the sense that for each phase, we have to characterize the evolution in terms of one process before doing so for the other one.

To select which process to start with, a choice motivated by mechanical expert judgments is made for each phase: There is a *cause-effect* relationship between the two indicators of each phase. For example, for the second and third phases, the corrosion current density is the *cause*, and the width of the crack and internal stress are the *effect*. When the corrosion current density increases, the tensile stresses on concrete also increases accelerating the concrete cracking initiation and propagation. At the same time, the presence of rust or cracks will change the material properties and have an effect on the corrosion current density (mutual dependencies).

As a consequence, to simulate the model we first seek to characterize the evolution in terms of the *causal* process then doing so for the respective *effect* process.

In Table 3.1, the indicators, their nomenclature and the identification of causal and effect processes are summarized.

The choices of each shape function is motivated by the evolution of its corresponding physical indicators over time, therefore we summarize in Figure 3.4 the mean simulations portraying the tendencies of each indicator. These tendencies will dictate later the mathematical formulation of the degradation model.

Table 3.1 Indicators of the pathology

Phase	Nomenclature	Physical indicator	Process' order
1st phase	$(\rho_{1,t})_{\forall t\geq 0}$	$[Cl^-]$ (%)	*causal*
	$(\theta_{1,t})_{\forall t\geq 0}$	Basicity of the concrete, *pH*	*effect*
2nd phase	$(\rho_{2,t})_{\forall t\geq 0}$	Internal tensile stress, σ (MPa)	*causal*
	$(\theta_{2,t})_{\forall t\geq 0}$	Corrosion current density, i_{corr} (μA/cm²)	*causal*
3rd phase	$(\rho_{3,t})_{\forall t\geq 0}$	Width of the crack, *a* (mm)	*effect*
	$(\theta_{3,t})_{\forall t\geq 0}$	Corrosion current density, i_{corr} (μA/cm²)	*causal*

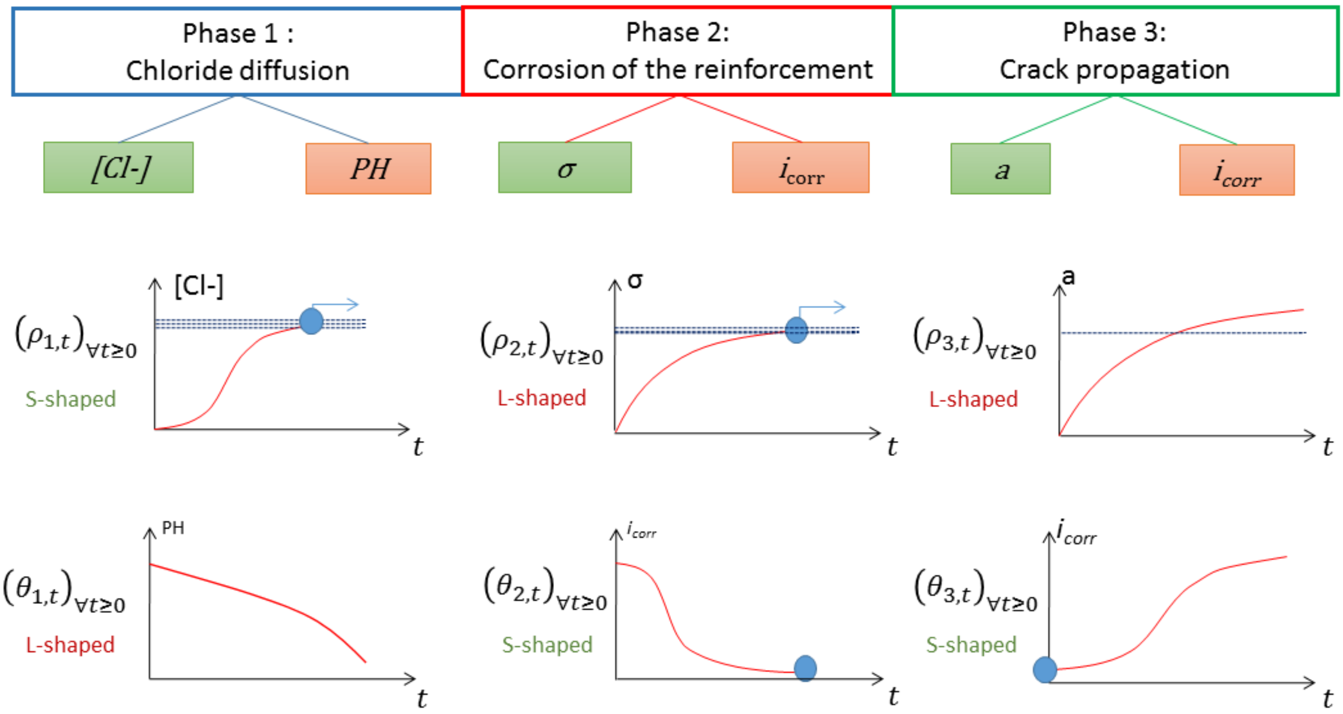

Figure 3.4 Tendencies of evolution of the degradation indicators

In Figure 3.4, we can note similar tendencies among the indicators:

- ρ_1, θ_2 and θ_3 share an S-shaped tendency in time.
- θ_1, ρ_2 and ρ_3 share an L-shaped tendency in time.

The shared tendencies between indicators (Figure 3.4) simplifies the mathematical formulation of each process since they will have similar mathematical expressions. In fact, the shape functions, that we aim to define here, control the state-based sizes of the increments who in their turn will control the tendencies of the physical indicators. Therefore, the similar tendencies of the physical indicators will generate similar mathematical expressions. On another level, this illustrates how modelling one pathology can benefit to model others with similar tendencies.

For example, for an S-shaped tendency, the increments start to increase at the start of the process, then after a defined level of degradation, start to decrease. This variation of increments is modelled using the shape function, and in this case, the increasing then decreasing of increments will form a bell-shaped shape function (Eq. 3.1). A similar observation is made for the L-shaped tendency where increments are bigger at the start of the process, then start to decrease with the level of degradation (Eq. 3.2).

With that said, here we summarize and then discuss the common shape functions for the three processes:

S-shaped state-dependant shape function: $$g'(y) \times e^{\frac{-(x-a_1)^2}{a_2}} \qquad 3.1$$

L-shaped state-dependant shape function: $$g''(y) \times e^{-a_3.x} \qquad 3.2$$

Where:

- $g'(y)$ and $g''(y)$ are the acceleration functions
- x is the level of degradation of the first indicator
- y it the level of degradation for the other indicator
- a_1, a_2, a_3 are the parameters of the shape function.

The exponential portions in equations 3.1 and 3.2 governs the variation of the increments function of x (*e.g.*, bell-shaped), that is Δx function of x. These exponential expressions is a choice, we might as well take different forms that respects the required tendencies.

On the counterpart in the equations, $g'(y)$ and $g''(y)$ governs the acceleration functions, i.e., the effect of one indicator on the other or the dependency between the indicators.

One of the objectives in using degradation meta-models is to minimize the number of parameters, therefore, the simplest type of dependency is linear. In this study, we are not aiming to optimise the dependency of the two processes, therefore, we consider $g'(y)$ and $g''(y)$ to be a linear function of state.

Using this uniformed approach, we can formulate the degradation model for the three phases of the pathology. However, to model the whole life time of the structure, we have to model the transition between phases. Therefore, the transition are modelled using thresholds on the indicators (discussed with more details in § 4.2.3).

In the next section, we limit the construction to the third phase only with the intention to illustrate the pedagogical use of the approach, and later in Chapter 4, the model is adapted to fit to the first two phases of the pathology, in order for the model to cover the life time, hence, be usable in a maintenance context.

3.3.2 Construction of the degradation meta-model for the third phase

A degradation model must cover the three phases of the degradation process. Using these functions, we can calibrate the tendencies of each indicator. However, in this section, we limit the construction to the third phase only. This section will serve as a pedagogic illustration of the construction and tools of the meta-model.

The cracking phase of the submerged concrete structure is then characterized by:

- $\left(\rho_{3,t}\right)_{\forall t\geq 0}$ modeling the width of the crack « a » (mm)
- $\left(\theta_{3,t}\right)_{\forall t\geq 0}$ modeling the corrosion current density « i_{corr} » ($\mu A/cm^2$)

As far as we know, no work has been published for studying the mutual dependencies of the two processes (corrosion current intensity and cracking) described previously. One main reason can be attributed to the lack of experiments for this particular phase. Another good reason is that it's virtually impossible to integrate the mutual dependencies in the available physics-based models. The extension of the degradation model to two processes can be very rewarding in terms of maintenance and inspection optimization, especially in terms of reliability of the data because of the diversification of the NDT techniques (Ploix et al. 2011; El Hajj et al. 2014) and flexibility in the inspection policy when costs or information quality are quite different (Schoefs et al. 2012).

Moreover, the time dependence of the two processes is non-stationary and the patterns can be considered as state-dependent: the evolution laws depend on the level of the current degradation. Finally, these two variables form a bivariate cumulative process where the two sub-processes are mutually dependent.

The construction of the dependence of the two sub processes is motivated by expert judgments on the mechanical process; there is a cause-effect relationship between the corrosion current density and the width of the crack, when the corrosion accelerates (corrosion current density increases), the material undergoes more stress, which will be translated as an acceleration of the cracking. Consequently, the crack opening changes the material properties, inducing more oxygen and humidity near the steel, thus, affecting the corrosion current density.

Furthermore, one of the aims of using degradation meta-models is to minimize the number of parameters, a simple form for modelling the dependency between the two processes is using a linear function (Linear parts in equations 3.5 and 3.6). Later, it was shown to be enough to model the dependencies and to respect the acceleration tendencies of the bivariate process.

The correlation will be modelled in terms of mutual acceleration effects directly in each of the shape parameters of the gamma distributions. This model is sequential in the sense that in a first step we seek to characterize the evolution in terms of changes in the corrosion current density (Equation 3.3) before doing so for the cracking itself (Equation 3.4).

Finally, the advantage of a state dependent approach is that it transforms the process into a stationary one (with respect to time). We can then write, $\forall(\rho,\theta) > 0$:

$$\Delta\theta_3(\tau_3\,;\,\rho_3,\theta_3) \sim gamma\left(\alpha_{\theta_3}(\rho_3,\theta_3).\tau_3\,,\beta_{\theta_3}\right) \quad 3.3$$

$$\Delta\rho_3(\tau_3\,;\,\rho_3,\theta_3,\Delta\theta_3) \sim gamma\left(\alpha_{\rho_3}(\rho_3,\theta_3,\Delta\theta_3).\tau_3\,,\beta_{\rho_3}\right) \quad 3.4$$

The choice for each shape function is motivated by the evolution of the respective process in time (Figure 3.2 and Figure 3.3). In other terms, the S-shaped condition state evolution of the corrosion current density (Figure 3.2) requires a bell-shaped shape function, similarly, the L-shaped condition state evolution of the crack width (Figure 3.3) requires an akin shape function. As a result, we propose the following shape functions, $\forall(\rho,\theta) > 0$:

$$\alpha_{\theta_3}(\rho_3,\theta_3) = (c_3.\rho_3 + c_4).e^{\frac{-(\theta_3-c_1)^2}{c_2}} \quad 3.5$$

$$\alpha_{\rho_3}(\rho_3,\theta_3,\Delta\theta_3) = \left(c_6.\left(\theta_3 + \frac{\Delta\theta_3}{2}\right) + c_7\right).e^{-c_5.\rho_3} \quad 3.6$$

The exponential part of the shape functions ensures the required shape of the shape-function. The linear functions before the exponential parts serves for two roles: a) it allows the modelling of the dependencies of the two processes, and b) it plays an acceleration role.

We notice in Equation 3.6 the expression $\theta_3 + \frac{\Delta\theta_3}{2}$. In fact, this expression accounts for the mean value of the corrosion current density over the interval τ_3. In other words:

$$\frac{(\theta_3)_t + (\theta_3)_{t+\tau_3}}{2} = \frac{\theta_3 + (\theta_3 + \Delta\theta_3)}{2} = \frac{2\times\theta_3}{2} + \frac{\Delta\theta_3}{2} = \theta_3 + \frac{\Delta\theta_3}{2}$$

Now that the model has been defined, a physical meaning can be given to each parameter. In fact, the mathematical formulation of the model allows the identification of physical tendencies associated to the parameters.

In Table 3.2, physical meanings to each parameter are summarized.

These definitions can aid to better understand the model. The physical meaning given to the parameters helps later in chapter 4 (§ 4.3.2) to formulate the modelling of the effect of a maintenance action on the model; if a maintenance action accelerates the degradation or translates the inflection point in example, we know which parameter(s) is(are) responsible for the effect.

Table 3.2 Definition of parameters

Parameter	Definition
β_θ, β_ρ	Proportionality factors common to different structures (materials, *etc.*)
c_1	Abscissa of the inflection point of the corrosion current density
c_2	Reflects the dispersion around the inflection point
c_3, c_6	Acceleration coefficients
c_4	Speed at the origin of the corrosion current density
c_5	Reflects the kinetics of the process ρ
c_7	Crack growth rate at the origin

To illustrate the model, we propose in Figure 3.5 and Figure 3.6 to plot the shape functions of both processes. Furthermore, in Figure 3.7 simulations of 4 trajectories using the bivariate SDGP are presented where we can find the tendencies of the two represented indicators.

The following parameters are used:
$c_1 = 1$, $c_2 = 1$, $c_3 = 1$, $c_4 = 1.2$, $c_5 = 0.8$, $c_6 = 1.8$, $c_7 = 2$, $\beta_\rho = 0.3$, $\beta_\theta = 0.3$.

NB – in this chapter, these parameters are used in all cases except in cases where we define a new set of parameters. For these latter cases, the choice to change parameters is motivated by two reasons: a) to demonstrate that the model's simulation is not reserved for a particular case, b) to examine the response of the model to different set of parameters, in other words, to examine the effect of the parameters on the simulation of the degradation.

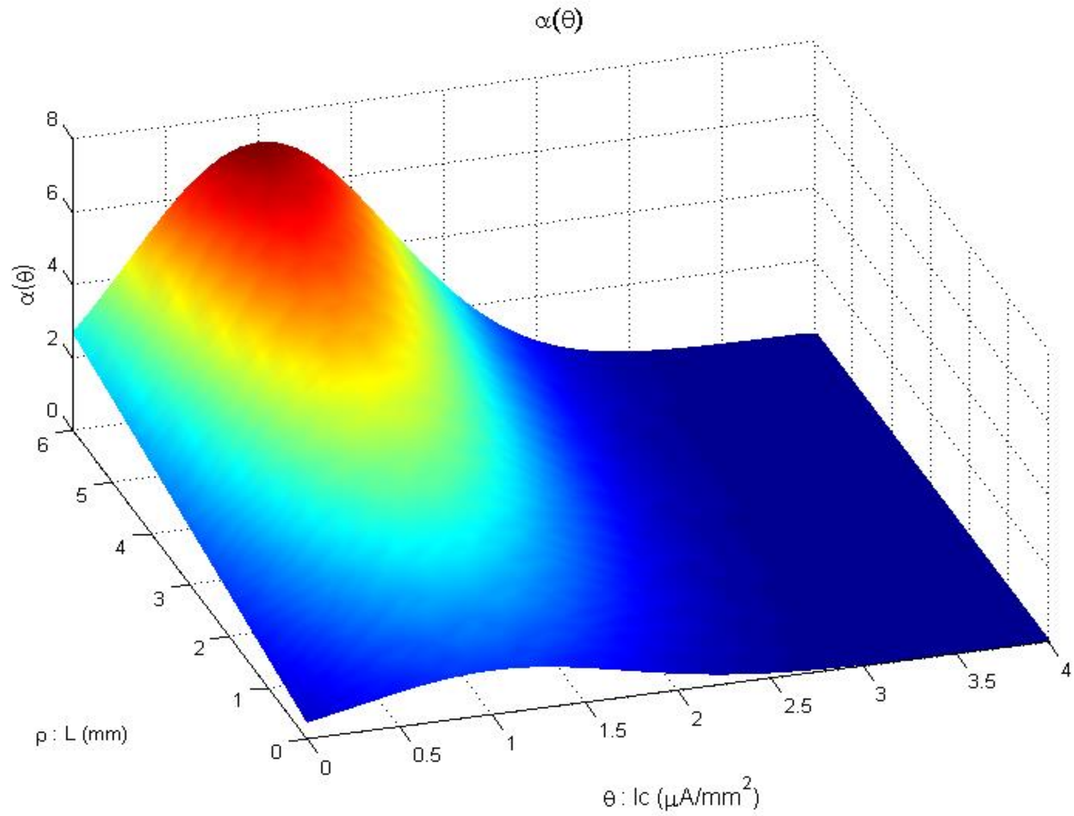

Figure 3.5 Shape function of the θ process - $\alpha_\theta(\rho, \theta)$

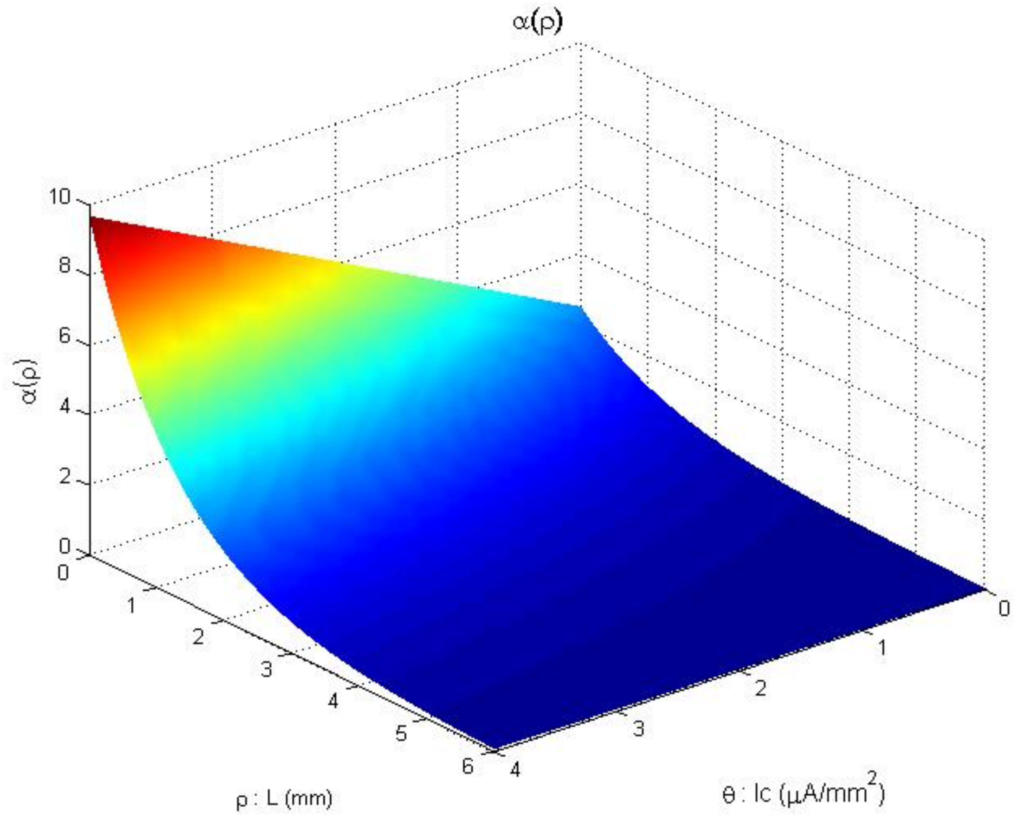

Figure 3.6 Shape function of the ρ process for constant Δθ - $\alpha_\rho(\rho, \theta, \Delta\theta)$

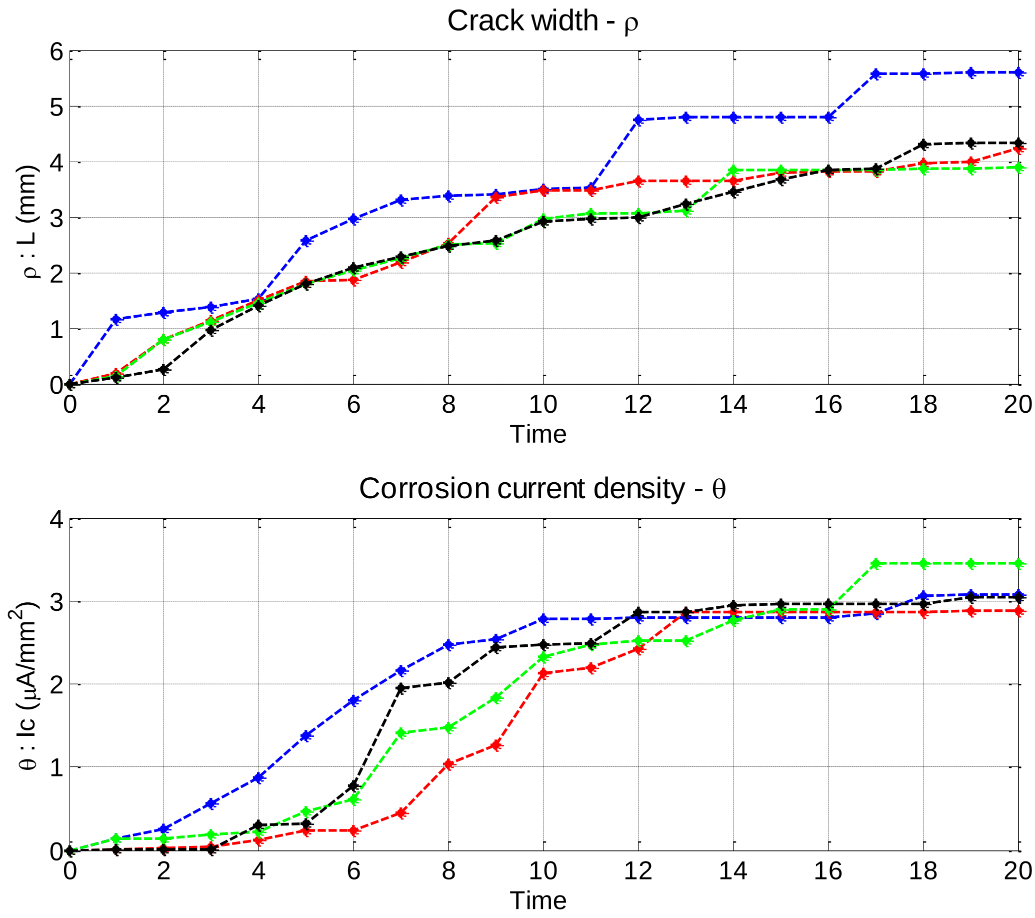

Figure 3.7 Example of 4 simulations using the bivariate SDGP

3.3.3 Databases description and estimation procedures

The database is considered to be formed from inspections carried out on statistically independent structures (same environment, same concrete and reinforcement design). Two values define the size of a database: the number of structures n, and the number of inspections carried out on each one of these structures T. Note that in this study, for simplicity, no spatial correlation is accounted for: that implies, for example, that n represents a set of structures (piles or beams of a given bridge, quay,…) in the same environment built with the same materials or a set of independent components (beams) on a given structure (O'Connor and Kenshel 2013; Schoefs, Clément, and Nouy 2009; Schoefs et al. 2014).

The database is assumed to contain measurements of the crack width and the corrosion current density of each *j* structure denoted $\left\{\left(\rho_t^{(j)}, \theta_t^{(j)}\right), t \geq 0, j \in [\![1, n]\!]\right\}$. Increments are directly computed using simple subtraction, and the resulting database group used in the estimation of the parameters of the bivariate state-dependent stochastic process are:

$$\left(\rho_t^{(j)}, \theta_t^{(j)}, \Delta\rho_t^{(j)}, \Delta\theta_t^{(j)}, t \geq 0, j \in [\![1, n-1]\!]\right)$$

A database can be constructed from periodic inspections at a fixed time step τ on statistically independent structures, here we talk of a complete database where the size of the database is given by:

$$N = n \times T \qquad 3.7$$

When a database contains N values over N, the database is complete. In this case, the estimation process is founded on a heuristic based on the classical MLE method, presented in sub-section 3.3.3.1.

However, values may be missing from the database for many reasons (§ 2.3.3), in such cases, we talk about incomplete databases where the total number of available values is smaller than N. An estimation algorithm based on an SEM algorithm is presented in sub-section 3.3.3.2.

3.3.3.1 Case of complete databases

We propose here to estimate the parameters of the degradation model using the MLE method on the existed complete database. However, because of the complex form of the shape functions, the estimation of the 9 parameters leads to numerical instability when using conventional optimization procedures. To work around this problem, we built a heuristic based on the fixed point theorem. This heuristic is applied iteratively to provide estimates of the parameters of the model (Equations 3.3 and 3.4).

The Heuristic is in two parts: The first part lies in the construction of the database samples, and calculating an initial β to initiate the fixed-point algorithm. The second part lies in the iterative estimation of the parameters of the shape functions at the jth, $\widehat{c_i}^{(j)}$, then calculate the scale parameters , $\hat{\beta}^{(j)}$. The iterations stop when the difference between two consecutive estimated $\hat{\beta}^{(j)}$ is smaller than a threshold ε.

To conclude, the Heuristic goes as follows:

Step 1:

i. *Construction of the database* $\left\{\left(\rho_t^{(j)}, \theta_t^{(j)}\right), t \geq 0, j \in [\![1, n]\!]\right\}$ *(resp.* $\left\{\left(\rho_t^{(j)}, \theta_t^{(j)}, \Delta\theta_t^{(j)}\right), t \geq 0, j \in [\![1, n]\!]\right\}$ *with* $\Delta\theta_t^{(j)} = \theta_{t+1}^{(j)} - \theta_t^{(j)}$*)*

ii. *Calculate the likelihood of Equation 3.3 (resp. Equation* 3.4 *) for the corresponding database*

iii. *Initiate* $\beta_\rho = \beta^{(0)}$ *(resp.* β_θ*)*

Step j, While $\left|\hat{\beta}^{(j)} - \hat{\beta}^{(j-1)}\right| > \varepsilon$, do:

iv. *Determinate the MLE estimates* $\widehat{c}_i^{(j)}$ *for* $\beta_\rho = \hat{\beta}_\rho^{(j-1)}$ *(resp.* β_θ*)*

v. *Evaluate* $\hat{\beta}^{(j)}$ *as the estimator of the MLE for the just considered* $\widehat{a}_i^{(j)}$.

End.

A study presented in the section 3.4.1 aims to analyse the convergence of this algorithm. In section 3.4.2, the same algorithm is used to test the benefit from including heterogeneous databases in the estimation process (§ 2.3.3.2).

Note. — We will not demonstrate the convergence of this fixed-point type algorithm. However, the large number of numerical experiments that we describe below portends the good properties of this algorithm.

3.3.3.2 *Case of incomplete databases (SEM algorithm)*

Incompleteness of databases is common in civil engineering. For many reasons (§ 2.3.3), databases exhibit to be incomplete by losing inspected values or not inspecting values. Causes of incompleteness vary from inspections techniques, measurement error (Schoefs, Abraham, and Popovics 2012; Torres-Luque et al. 2014), accuracy of the machines and cancelled inspections (due to security concerns, weather, costs, no available technicians…). Data that is lost can be classed into three categories; those are truncated, censored and missing:

i. Censored values are those reported as less than some value – left censored (*e.g.*, <5 cm), greater than some value – right censored (*e.g.*, > 0.1 µA/cm2), or as an interval – interval censored (*e.g.*, a value between 67 and 75 days).
ii. Truncated values are those that are not reported if the value exceeds some limit – right truncated (*e.g.* If the crack width is above 3 cm we stop recording) or if values exceed the physical understanding (*e.g.*, corrosion depth of steel more than original thickness).
iii. Missing data (§ 2.3.3.1) are when values are lost due to recording interruptions related to field data measurement or missed inspections.

All three categories occur frequently in civil engineering. Since missing data can have a significant effect on the drawn conclusions, it is important to take such limitations into consideration. In a condition-based maintenance context, every decision is based on the degradation level and therefore it is really important to give the best available prediction.

Essentially, the way we deal with the three types of incompleteness in databases is the same. Next, we will present the imputing and estimation algorithm used in the estimation process in the presence of missing data or incomplete database. An illustration of the algorithm using some numerical examples for the third case of missing data is presented in section 3.4.

A high level of censoring or missing data increases the numerical instability of the optimization problem in the estimation process, especially in the maximization of the likelihood. The Expectation-Maximization (EM) algorithm, introduced by (Dempster, Laird, and Rubin 2007), is an iterative procedure designed to find maximum likelihood estimates in the context of parametric models where the observed data can be viewed as incomplete. The EM algorithm is a simple approach; unfortunately it has its limitations. As noted in (Dempster, Laird, and Rubin 2007), the convergence rate of EM is linear and governed by the fraction of missing information meaning the EM algorithm can be extremely slow when the proportion of missing data is high. Moreover, the EM is proved to converge to a stationary point of the log-likelihood function, but when several stationary points are present, the algorithm does not necessarily converge to a significant local maximum of the log-likelihood function.

Due to the high number of parameters in our model, the stochastic nature of the process, and the presence of incomplete database, we need to extend the EM algorithm.

To answer to the limitations of the EM algorithm, the SEM algorithm was proposed by (Celeux and Diebolt 1985), where the S stands for stochastic. The SEM algorithm incorporates a stochastic step between the E-step and the M-step of the EM algorithm. The Stochastic step is based on the Random Imputation Principal (RIP) meaning: to replace each missing quantity with a value drawn at random from the conditional density $q\left(y \middle| x, \theta^{(n)}\right)$ where $\theta^{(n)}$ is the parameter estimate at n-th iterations, y denotes the missing data, x denotes the observed data and z denotes the complete database: $z = x \cup y$.

The SEM n-th iterations are as follows:

- E-step: compute the conditional density $q\left(y \middle| x, \theta^{(n)}\right)$
- S-step: using RIP, we simulate the unobserved data to draw a complete sample $z^{(n)}$.
- M-step: find the ML estimates of $\theta^{(n+1)}$ using the complete sample $z^{(n)}$.

The program starts by scanning the provided database for potentially missing data, and indexes them. Then, using the observed data, initial parameters are estimated and used to start the iterative modified SEM algorithm.

In the S-step we include a test that verifies if the generated data respect the positive increments condition. For the M-step, and because of the high number of parameters in $\theta^{(\mathrm{n}+1)}$, an iterative two-step heuristic algorithm is used for the maximization of the log-likelihood. The selected shape functions lead to numerical instability problems with conventional optimization procedures. To circumvent this problem, we constructed a heuristic based on the fixed point theorem. This heuristic is applied iteratively to provide estimates of the parameters of the model (Equations 3.3 and 3.4).

In Figure 3.8, the SEM algorithm is presented.

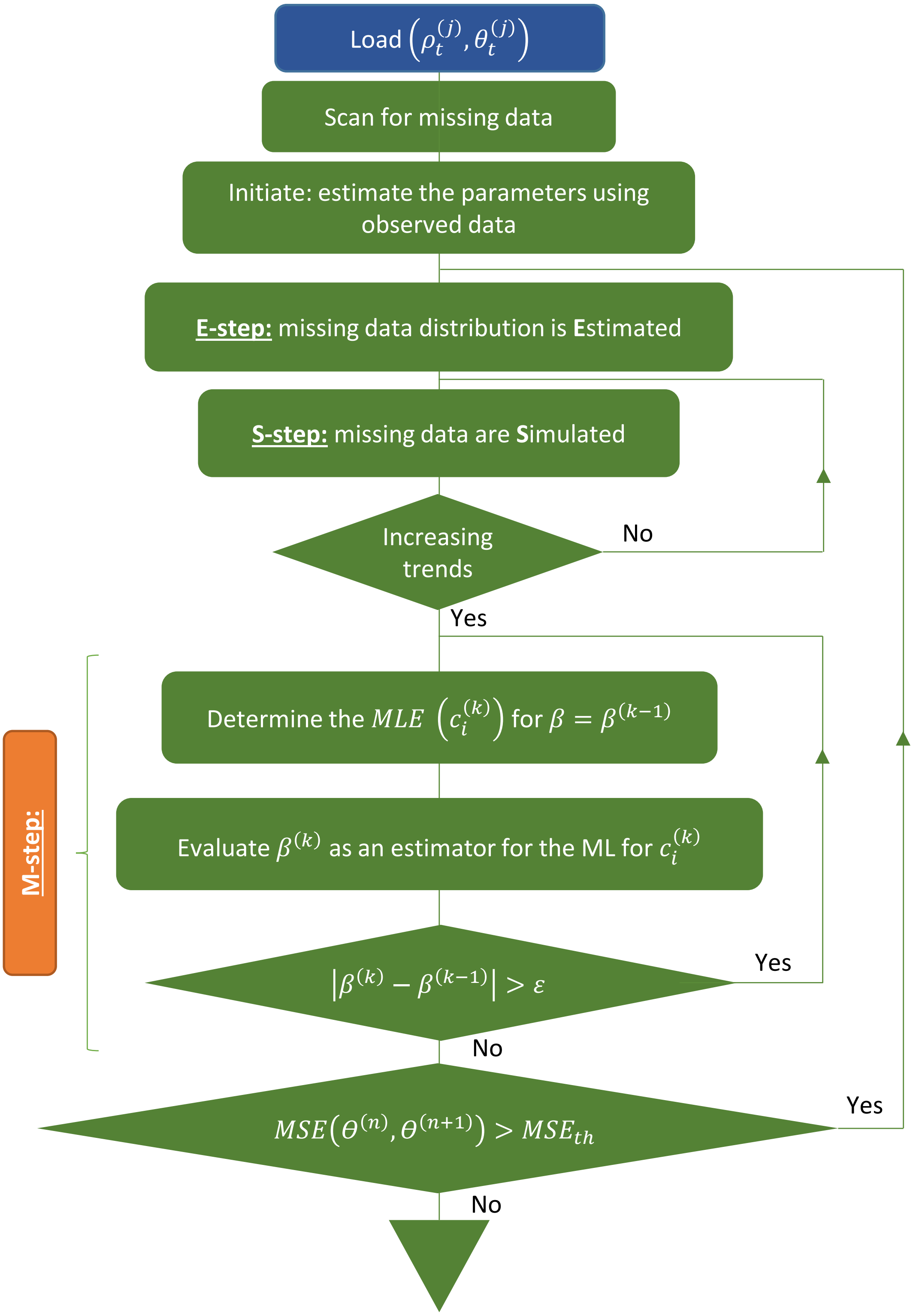

Figure 3.8 Stochastic Estimation-Maximization algorithm

The SEM iterative algorithm stops for a desired accuracy of the estimated parameters: this accuracy is expressed in terms of target mean square error (MSE), we select here a threshold $MSE_{th} = 0.1$.

Note. — We will not demonstrate the convergence of this fixed-point type algorithm. However, the large number of numerical experiments that we describe below portends the good properties of this algorithm.

3.4 Numerical experiments in irregular contexts

It is a fact that in civil engineering it is very unlikely to find a complete database that doesn't suffer from at least one sort of irregularity, whether intentionally or by accident. Because of that, in a practical and realistic context, it is primordial to take consideration in degradation models for irregularities in databases even before encountering them. Moreover, the potential benefits, in terms of design and maintenance management, from using information issued from heterogeneous databases emphasizes on the need to allow to degradation models to include such information.

In this section, we propose to conduct a set of numerical analyses to highlight the statistical inference of the model properties especially its estimation abilities.

To that aim, first, in section 3.4.1, the convergence of the basic estimation process for a complete database is verified. Then, in section 3.4.2, the benefit of using heterogeneous databases in the estimation process is studied. And finally, in section 3.4.3, we investigate the SEM imputing algorithm that deals with censored and missing values from the databases, furthermore, we examine the effect of missing data and censored data on the estimation process.

However, for the third phase of degradation, no real field databases were available for us to use. Hence, we propose to simulate a set of virtual databases. In this study, we consider different types of databases (complete, heterogeneous, missing data and censored). In each case, we will detail the method used to include irregularity in the databases.

The Mean Squared Error (MSE) is used as the performance criteria for the estimation process. To illustrate the performance of the estimation process and its convergence, we propose to evaluate the MSE of the estimated parameter vector $\hat{\Theta}$ based on the simulated database given the true parameter values θ^*:

$$MSE = trace\left\{E\left\{\left(\hat{\Theta} - \Theta^*\right)\left(\hat{\Theta} - \Theta^*\right)^T\right\}\right\} \qquad 3.8$$

Where the true parameters are grouped in $\Theta^* = \{c_1{}^*, c_2{}^*, c_3{}^*, c_4{}^*, c_5{}^*, c_6{}^*, c_7{}^*, \beta_\theta{}^*, \beta_\rho{}^*\}$, and the estimated parameters in $\hat{\Theta} = \{\widehat{c_1}, \widehat{c_2}, \widehat{c_3}, \widehat{c_4}, \widehat{c_5}, \widehat{c_6}, \widehat{c_7}, \widehat{\beta_\theta}, \widehat{\beta_\rho}\}$.

In this example, we use the same parameters used in § 3.3.2 for Θ^*, these are:
$c_1 = 1$, $c_2 = 1$, $c_3 = 1$, $c_4 = 1.2$, $c_5 = 0.8$, $c_6 = 1.8$, $c_7 = 2$, $\beta_\rho = 0.3$, $\beta_\theta = 0.3$.

3.4.1 Convergence of the estimation procedure for complete database

Here, we consider that the database is constructed from periodic inspections at a fixed time step τ on independent but identical structures or parts of the same structure. Two values define the size of the database N: the number of inspected structures n, and the number of inspections carried out on each one of these structures T.

Table 3.3 summarizes the MSE on the estimations for each one of the 9 parameters separately, per process, MSE_θ and MSE_ρ, and for the global process MSE_t. In Figure 3.9, three curves of MSE_t for three values of n are illustrated as a function of T.

Table 3.3 MSE in the case of a complete database

T	n	$n*T$	c_1	c_2	c_3	c_4	β_θ	MSE_θ	c_5	c_6	c_7	β_ρ	MSE_ρ	MSE_t
5	5	25	0,52	246	0,48	0,85	0,003	247	0,60	236	780	0,0029	1011	1258
	10	50	0,03	5,44	0,19	0,46	0,002	6,1	0,06	5,19	0,22	0,0015	4,90	11
	20	100	0,01	0,05	0,09	0,22	0,001	0,38	0,03	1,23	0,10	0,0007	1,28	1,7
	50	250	0,01	0,03	0,05	0,07	0,000	0,16	0,01	0,47	0,19	0,0003	0,65	0,8
10	5	50	0,05	0,12	0,36	0,64	0,003	1,15	0,04	4,09	0,43	0,0020	4,06	5,21
	10	100	0,08	0,12	0,33	0,32	0,002	0,80	0,02	1,42	0,57	0,0010	1,91	2,72
	20	200	0,02	0,04	0,09	0,20	0,001	0,33	0,01	0,55	0,08	0,0005	0,61	0,95
	50	500	0,02	0,03	0,05	0,09	0,000	0,15	0,003	0,24	0,1	0,0002	0,33	0,5
20	5	100	0,16	0,22	0,67	0,45	0,004	1,35	0,01	1,64	0,98	0,0018	2,53	3,9
	10	200	0,08	0,15	0,25	0,34	0,002	0,75	0,006	0,67	0,24	0,0008	0,86	1,62
	20	400	0,04	0,09	0,15	0,23	0,001	0,45	0,003	0,28	0,09	0,0004	0,35	0,8
	50	1000	0,02	0,05	0,06	0,13	0,001	0,19	0,001	0,10	0,03	0,0002	0,13	0,3

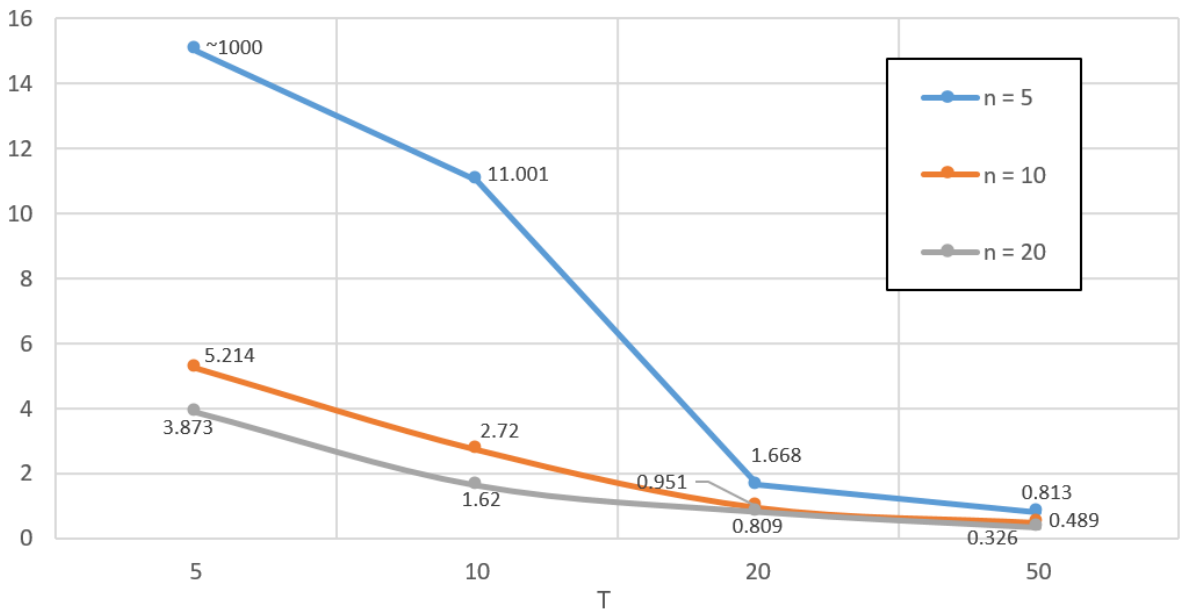

Figure 3.9 Total Mean Squared Error (MSEt)

These results illustrate the convergence of the basic estimation heuristic, with a strong improvement for $T > 20$ and $n > 10$. Furthermore, we want to investigate the quality of the estimation based on the number of structures to monitor n and on the observation time per structure T.

Before going further into the analysis of these results, we remind that the number of data needed to obtain a good estimate may appear important, however, we recall here that we are interested in estimating laws for an entire life cycle of a structure in which the number of parameters remains quite significant.

From Table 3.3, we note a variety of behaviours depending on the parameters. We principally retain that the procedure favours the estimation of scale parameters and the parameters of the shape functions directly related to the level of cracking. This is mainly observed for samples of very small size. This can be explained by the fact that the corrosion current density level at $t = 0$ at the initiation of the crack is a random variable, varying within a given range while the value at the origin of ρ is fixed. It is therefore difficult to calibrate the tails of the distributions when the only observations are concentrated in the early life cycle ($T = 5$) with a small number of structures ($n = 5$).

To conclude this section, one can measure the compromise between extending the observation period and increasing the number of structures to monitor; we then take for a sample size $n \cdot T$ with different n and T and we compare the MSE_t (see columns 3 of Table 3.3). From these results, we observe a very fast compensation when adding new structures. This point is particularly interesting in the sense that it is often easier and better from decision aid tool to find analogous structures than wait for the next inspection. We speak of heterogeneous databases, and in the next section we investigate further this hypothesis.

3.4.2 Benefit of considering heterogeneous database

In section 2.3.3.2 we discussed heterogeneous databases and the potential benefit of using them in degradation understanding and modelling. Here we test this hypothesis by analysing the adaptability of the meta-model to heterogeneous databases, first, from a parameter inference point of view, and then, from a reliability point of view.

Degradation parameters (cracking initiation, crack growth rate) are related to specific properties inherent to the studied structure. In civil engineering, it is clear that it can be difficult to qualify databases samples as homogenous for many reasons such as the strong heterogeneity in materials, formulas, conception processes, service conditions, environment, and previous maintenance actions. Therefore, the consideration of solely homogenous databases may result in the selection of samples with very small sizes, this phenomenon is also amplified with the poorness of current database and the quality of the inspection policy (Khraibani 2010). As a consequence, the quality of the estimation process can be strongly degraded due to the lack of data.

Motivated by the fact that the degradation models here are state-dependent, we tend to feel that each available data can provide valuable information to the model. Each unique data, in its individuality, may help the construction of the model on the global scale where each state gives input to build the continuous function of distribution of the state-dependent increments. In this context, first we test the estimation procedure when combining heterogeneous databases, then, we compare classical reliability parameters function of heterogeneity.

3.4.2.1 Robustness of the estimation procedure in respect to the heterogeneity of the database

In this study, we want to investigate the robustness of the proposed estimation algorithm when faced with heterogeneity in a database. For this, we propose to generate databases with N trajectories (from the crack initiation to the end of life for each of the N structures). In each database, the heterogeneity is obtained by integrating some controlled random variability on the given parameters of the proposed bivariate meta-model. Hence, for $N = 20$, 5 of the trajectories are simulated with the original parameters (no variability), 5 more with 5% variability, 5 more with 10% and the last 5 with 15%. And then for $N = 15$, 5 homogenous trajectories, 5 more with 5% variability and 5 with 10%. For $N = 10$ we have 5 homogenous trajectories and 5 trajectories with 5% variability. And finally $N = 5$ formed by 5 homogenous trajectories.

The variability sketches the heterogeneity of the database and the challenge is in improving the homogeneity of the database and is illustrated by the decreasing from $N = 20$ structures to $N = 5$ (perfectly homogenous case) for the estimation of the model parameters $\{\hat{c}_i, i = 1, \ldots, 9\}$ by removing 5 structures each time.

We propose to analyse the benefit of including more structures, even heterogeneous ones, in the estimation process using the MSE as a measure. Table 3.4 contains the mean of the MSE obtained for 500 databases, $\overline{MSE}$ (third row). The second row, $\overline{MSE}_0^{(N)}$ is the mean values obtained for homogeneous databases (no variation in the parameters for the whole simulated database) with different number of structures. The last row is the relative error given by the equation:

$$\bar{e} = \frac{\overline{MSE} - \overline{MSE}_0^{(N)}}{\overline{MSE}_0^{(N)}} \qquad 3.9$$

Table 3.4 Impact of the heterogeneity level of the sample on the estimation performance

N	5	10	15	20
$\overline{MSE}_0^{(N)}$	4.979	2.179	1.504	1.110
$\overline{MSE}$	4.979	2.502	1.518	1.223
$\bar{e}$	0	0.148	0.009	0.012

This numerical application emphasizes on the potential benefit to the estimation process when considering additional data even though they originate from strongly heterogeneous structures. In this example, the increase in the relative error when $N = 20$ suggests that the homogeneity level should be optimized to ensure the estimation quality.

To further test the hypothesis, we carry out the same calculation as before using a new set of parameters. Furthermore, we add one more case where 5 more trajectories with 20% variability are added to the database. The new set of parameters used for this test is:

$c_1 = 1$, $c_2 = 2$, $c_3 = 2$, $c_4 = 0.8$, $c_5 = 0.6$, $c_6 = 1$, $c_7 = 1.4$, $\beta_\rho = 0.3$, $\beta_\theta = 0.3$.

The reason behind choosing a new set of parameters here is to verify the validity of the previous conclusion for different degradation tendencies.

Table 3.5 contains the means of the MSE for two additional forms of variability: when considering only positive variability $\overline{MSE}_+^{(N)}$ and only negative variability $\overline{MSE}_-^{(N)}$. MSE are represented in bold, also, adjacent to the MSEs, the table contains relative errors using equation 3.9.

Table 3.5 Impact of the heterogeneity level of the sample on the estimation performance #2

N	5		10		15		20		25	
$\overline{MSE}_0^{(N)}$	**12.21**		**7.37**		**4.38**		**3.63**		**3.24**	
$\overline{MSE}$	**12.21**	0	**7.39**	<0.01	**5.01**	0.14	**3.80**	0.05	**3.39**	0.05
$\overline{MSE}_{-}^{(N)}$	**12.21**	0	**6.50**	-0.12	**5.07**	0.16	**4.25**	0.17	**2.61**	-0.19
$\overline{MSE}_{+}^{(N)}$	**12.21**	0	**6.65**	-0.10	**4.06**	-0.07	**3.72**	0.02	**3.48**	0.08
		$\bar{e}$		$\bar{e}$		$\bar{e}$		$\bar{e}$		$\bar{e}$

The same analysis done in Table 3.4 is used for Table 3.5. However, it can be intriguing to see that for some cases the introducing of heterogeneous databases lower the MSE, *e.g.* for $N = 10$, $\overline{MSE}_{+}^{(10)}$ and $\overline{MSE}_{-}^{(10)}$, where we have +5% and -5% respectively (small variability), are lower by almost 10% than $\overline{MSE}_0^{(10)}$. Similar observations can be made for $\overline{MSE}_{+}^{(15)}$ and $\overline{MSE}_{-}^{(25)}$. These results reflect the improvement of estimation process when considering heterogeneous databases, to a certain level, however, the MSE used as a performance indicator here ponders 9 parameters together, therefore, doesn't take into account the individual weight of each parameter.

3.4.2.2 Impact of the heterogeneity level on the reliability performance

In this section we illustrate the effect of variability on the parameters in terms of durability. A failure threshold on ρ is introduced, and the lifetime is defined as the number of inspections to failure. The heterogeneity in the sample is directly modelled in terms of variability on the parameters.

In Figure 3.10, we illustrate the probability distribution functions (PDF) of a failure at the kth inspection when the variability sweeps the following values: 0%, $\pm$10%, $\pm$20%, $\pm$ 30% and $\pm$40%.

These results are obtained from 25000 simulations, using a new set of parameters:
$c_1 = 2$, $c_2 = 4.5$, $c_3 = 1.8$, $c_4 = 1.8$, $c_5 = 0.65$, $c_6 = 1$, $c_7 = 1$, $\beta_\rho = 0.2$, $\beta_\theta = 0.2$.

The parameters are changed here to pronounce further the effect of the variability on PDFs.

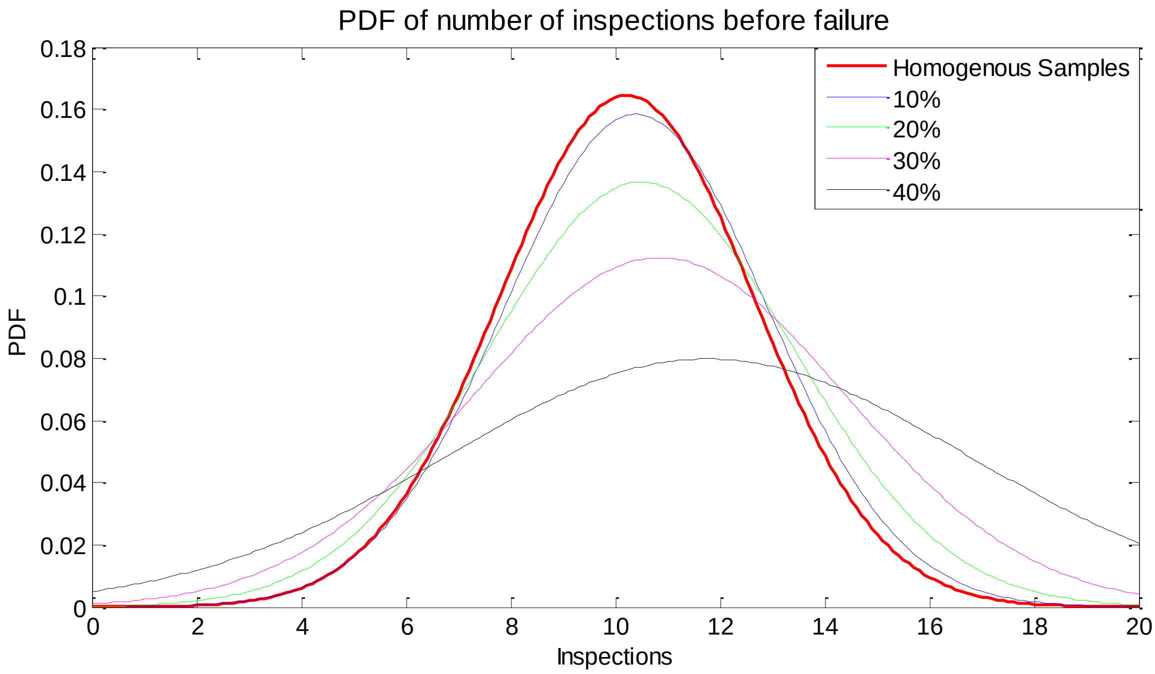

Figure 3.10 Distribution Density of a failure observed at the k^{th} inspection for different variability rate on the parameters

The heterogeneity level could impact the durability estimation in two ways. First, some bias is introduced in the mean lifetime estimation when the variability on the parameters increases. For decision-making, this effect would lead to a bad inspection policy. The second point is about the propagation of the uncertainty in the model illustrated in the increase of the variance of the lifetime distribution. Table 3.6 sketches the evolution of the associated relative increase in the standard deviation of the lifetime.

Table 3.6 Impact of variability on the statistical moments of the number of inspections before failure

	± 0%	±10%	±20%	±30%	±40%
μ	10.26	10.34	10.56	10.91	11.53
e_{μ} (%)		0.82	2.94	6.37	12.48
σ	2.43	2.57	2.96	3.68	4.78
e_{σ} (%)	0	5.51	21.52	51.15	96.16

On this numerical experiment, we can conclude that a homogeneity level around ±10% remains eligible for the estimation of the mean lifetime. Beyond this variability, the quantification of a structure in terms of durability is too hazardous.

3.4.3 Convergence of the SEM algorithm when data are missing

We distinguish between two types of missing data; a) simultaneously or "*total*" missing data: if in one of the processes ($\rho_{\mathrm{t}}^{(\mathrm{j})}$or $\theta_{\mathrm{t}}^{(\mathrm{j})}$) a data is missing, it is certain that data will be missing for the same inspection in the other process (respectively $\theta_{\mathrm{t}}^{(\mathrm{j})}$ or $\rho_{\mathrm{t}}^{(\mathrm{j})}$). b) Non- simultaneously or "*partial*" missing data: a more random case where the missing data can be unavailable from one process, but not necessarily from the second one. On the other hand, the right censoring (Type 1) is also considered, meaning that inspections will cease at predefined time (no more inspections after).

In this example, we will illustrate a general case in a context of randomly censored and *partial* missing data. We consider the case of $n = 15$ structures, we start by simulating n simulations with no missing nor censored data using the two state-dependent stochastic processes. For each simulation, 10 inspections per simulation are considered. Therefore, the full database is formed of 15 structures and 10 inspections each.

Now we aimlessly delete 25% of the data from each simulation. To delete 25% of the data, we use a uniform distribution over the recorded time to choose random inspections times to delete.

The censorship comes next, modelled using a normal distribution centred on the 10$^{\text{th}}$ inspection, with a variance $=\ 5$ and truncated to the right. Then we extract 15 numbers from this distribution and these numbers will be the censorship time after which all data shall be removed from the databases.

Figure 3.11 compares the average estimated behaviour of the structures from the incomplete database. In this case, the estimated parameters are estimated after 5 iterations of the SEM (under 20 seconds using an office PC), stopping the algorithm for a $MSE = 0.0905 < 0.1$.

In Figure 3.11 we can see the estimation from uncompleted and completed databases are really close to each other, and to the mean from the complete database. Moreover, the trends simulated in this figure re-join the experimental trends illustrated in Figure 3.2 and Figure 3.3.

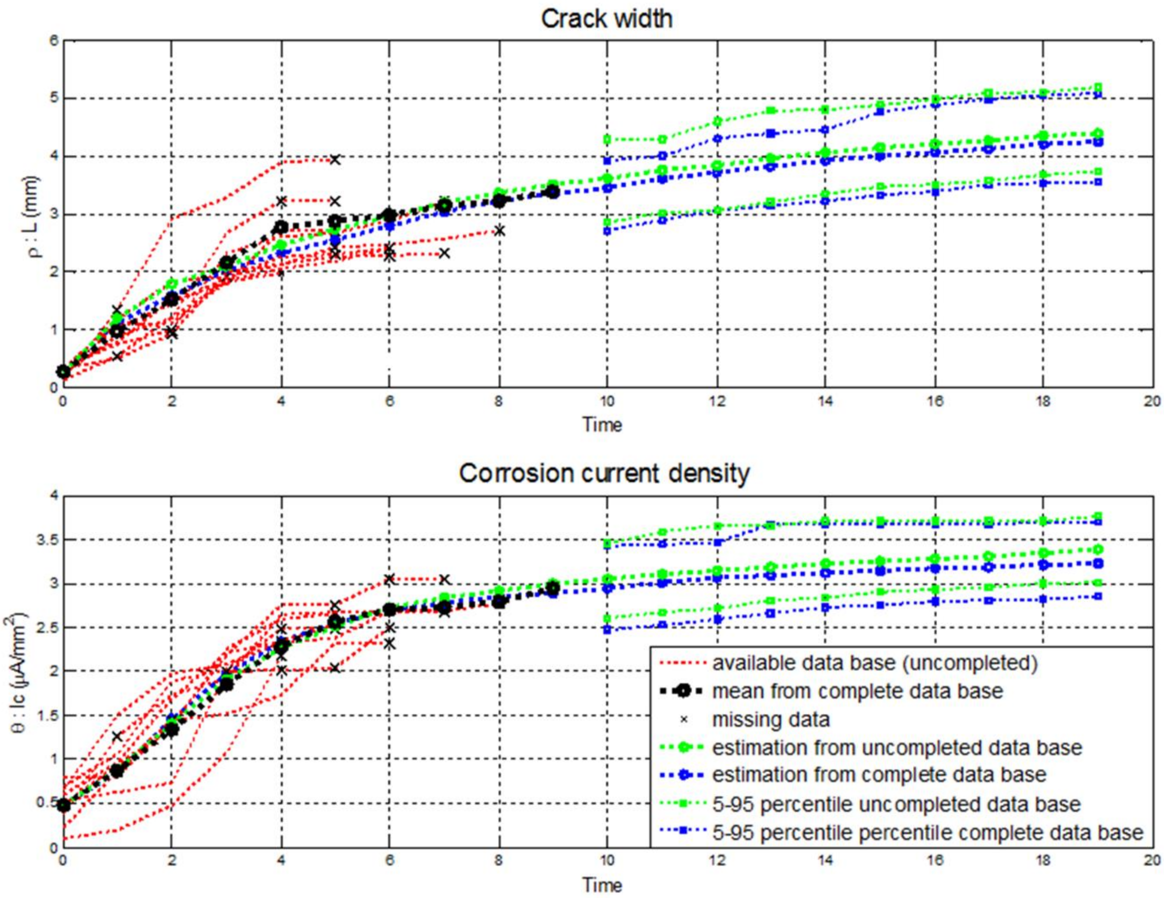

Figure 3.11 Estimation with randomly missing and censored data

Table 3.7 compares the parameters estimates for this example. In conform to Figure 3.11, the estimated parameters of the incomplete database are fairly close to the estimates of the complete one, and to the parameters used to create the original database.

Table 3.7 Numerical example parameters estimate

	c_1	c_2	c_3	c_4	β_ρ	c_5	c_6	c_7	β_θ
Database parameters	1	1	1	1.2	0.3	0.8	1.8	2	0.3
Full-database estimates	1.04	1.17	0.58	1.49	0.29	1.03	4.25	1.13	0.31
Missing data estimates	0.88	1.65	0.87	1.41	0.26	0.96	3.46	0.78	0.39

Figure 3.12 and Figure 3.13 illustrate the shape functions of ρ and θ for parameters estimated from incomplete data base. These plots re-join in terms of shapes (*i.e.*, bell--shaped for instance) the ones illustrated in Figure 3.5 and Figure 3.6.

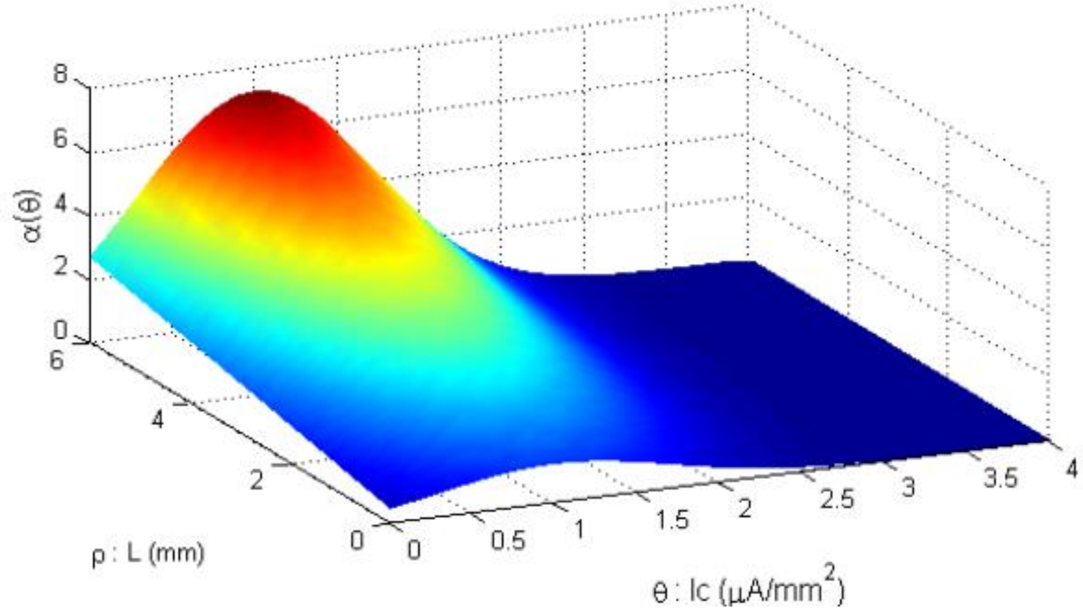

Figure 3.12 Shape functions of the θ process - $\alpha_\theta(\rho,\theta)$

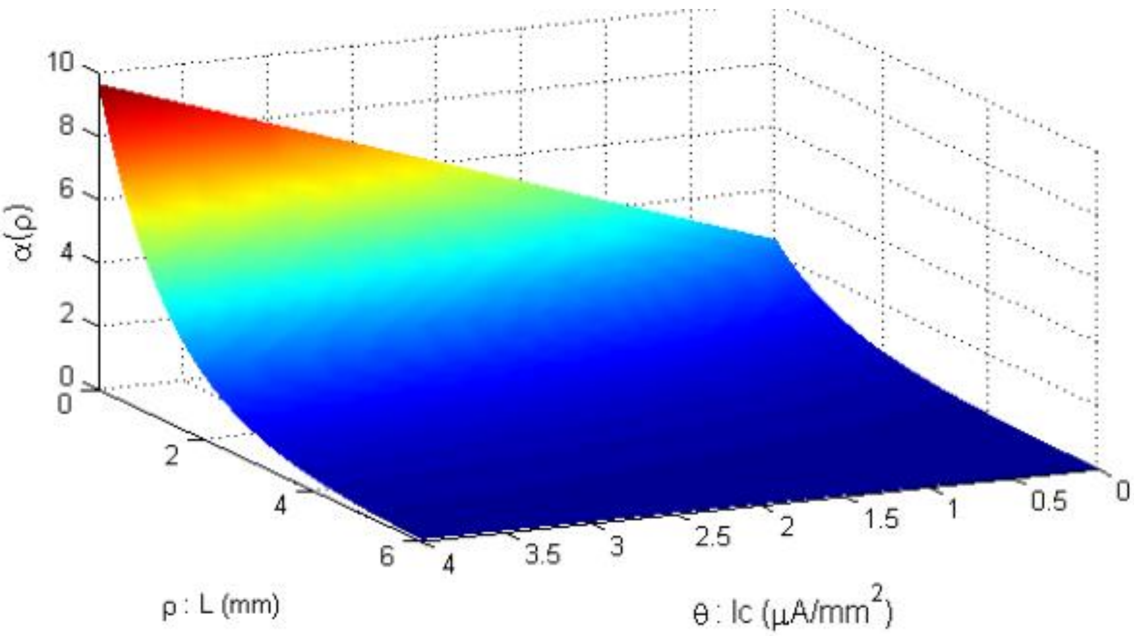

Figure 3.13 Shape function of the ρ process - $\alpha_\rho(\rho,\theta,\Delta\theta)$ for constant $\Delta\theta$ = 0.5

3.4.3.1 Effect of the level of missing data

Here, we are questioning the effect of the percentage of missing data on the estimates by means of MSE by independently considering the two cases: a) the **total** missing data and b) the *partial* missing data. We want to study the accuracy of our estimation algorithm as a function of the level and type of missing data over the lifetime.

We use a series of 10 inspections on n structures using the same model as in the previous section, then we randomly remove data from the database using a uniform distribution between the 1$^{\text{nd}}$ and the 10$^{\text{th}}$ inspection. We propose to vary this missing rate from 75% to 0%, where 0% corresponds to the use of the complete data set for the estimation.

The results are shown in Table 3.8. Each cell represents the MSE of the estimated parameters based on the simulation of 400 scenarios generated for the two types a) in **bold** in the table and b) in *italic*. The number of structures and the percentage of the missing data are the variables of this study. No censored data are considered in this study.

Table 3.8 MSE for Missing Data

Missing rate	75%		50%		25%		0%
Number of structures	**Total**	*Partial*	**Total**	*Partial*	**Total**	*Partial*	Full data-base
5	2.20	1.78	1.756	1. 7019	1.5275	1.4689	0.96
10	0.8782	0.8508	0.7371	0.6679	0.6284	0.5909	0.46
20	0.4231	0.3887	0.422	0.3765	0.305	0.3014	0.24

We notice that the MSE and n are inversely proportional, when n doubles the MSE is roughly divided by two. And clearly the estimation process is convergent. But, the aim of this section is to study the effect of the missing data on the accuracy of the estimation process, therefore it is important to explain for instance the significance of an MSE equals to 2.2, and if it is a good or a bad estimate. Therefore, we propose to plot six random results of the estimation on one graph with their MSE. We consider the following cases: 3 results: $n = 10$ with 50% missing data rate, then 2 results: $n = 10$ with 75%, followed by one result from the worst case scenario, $n = 5$ and 75%. The results are illustrated in the Figure 3.14.

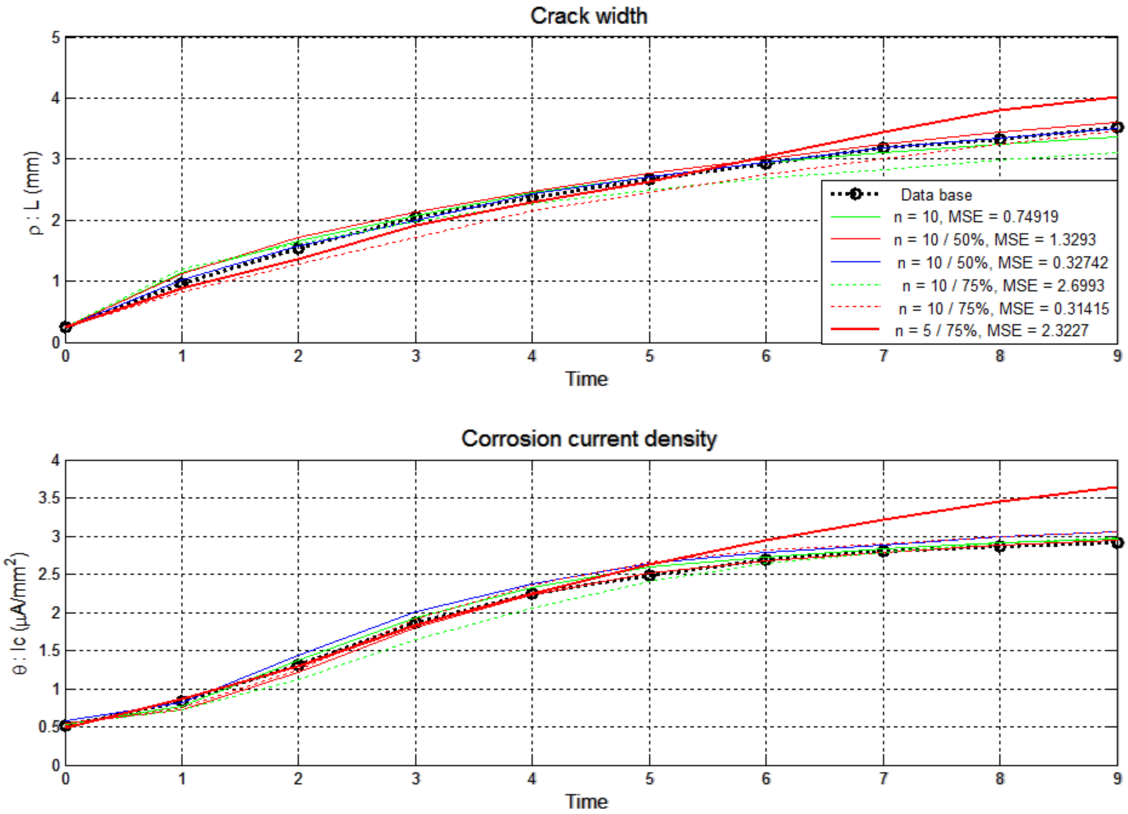

Figure 3.14 Means simulations

From Figure 3.14, an $MSE = 2.3227$ over estimates the degradation (red line). It is important to remind that this result is for 5 structures with a large 75% missing data rate, meaning we use approximately 12 points to estimate the 9 parameters.

On another hand, this is a multi-parametric problem (9 parameters), and clearly a complex one; it is possible to have different sets of parameters giving the same simulations. In example, in Table 3.7 we compute the MSE for the two estimated sets and we get:

$$MSE_{full} = 0.79 > MSE_{missing} = 0.53$$

However, the simulation using the full database estimates will be better since it maximizes the likelihood function. Consequently, one might question the efficacy of using MSE in this context; the reply would be that by using 400 simulations, the results will balance out when comparing with the same estimation model, n and missing rate.

Another interesting result is the difference between partial and total missing data. We notice that the MSE in the partial case is slightly smaller than the MSE in the total case. The most significant difference is in the worst case scenario of 5 structures and 75% missing data rate where the MSE drops 0.4.

3.4.3.2 Effect of censored data

Here we aim to study the effect of the censored data on the estimates for the same number of potential inspections and the same model as in the previous sections.

For this purpose, we use a series of 10 inspections on *n* structures using the same model as in the previous section, then we randomly censor the data base differently for every structure. We propose censorship on the data from 0% to 80%, where 80% means that it is possible to have only 2 inspections for one structure and 0% meaning no censorship is applied. A random censorship level is modelled using a normal distribution centred on the last inspection (10th) and truncated to the right. Then we vary the variances respectively with the desired censorship and we generate the censorship times.

The results are shown in Table 3.9. Each cell represents the MSE of the estimated parameters based on the simulation of 400 scenarios. The number of structures and the percentage of the censored are the variables of this study.

Table 3.9 Effect of censored data and number of structures on MSE

Censored Data / Number of structures	80%	40%	0%	Full database
5	3,07	2,52	1,47	0,96
10	1,37	0,81	0,59	0,46
20	0,32	0,32	0,30	0,24

For this table we can have similar observation to the one obtained in the previous section: especially, increasing the number of structures offers a valuable assistance even in case of high levels of censored data. We can note here the impact of the first inspection (from the 80% case) in the case of a fair number of structures in giving good estimates.

3.5 Applications

In this section, we propose to conduct further numerical analyses to highlight the statistical inference of the model properties. First, in 3.5.1, we propose a study to highlight one of the benefits of using two indicators instead of one in terms of quality of prediction. Then, in 3.5.2, we use the meta-model in a risk-management framework where we discuss the model and potential decision-making tools.

3.5.1 Application to Eurocode 2

The meta-model is based on a bivariate degradation model. We propose here to highlight the benefits of the introduction of the θ-process in terms of quality of prediction and propagation of uncertainty in the overall cracking process. In Figure 3.15, the dotted lines correspond to one possible history of the cracking structure extracted from the generic database.

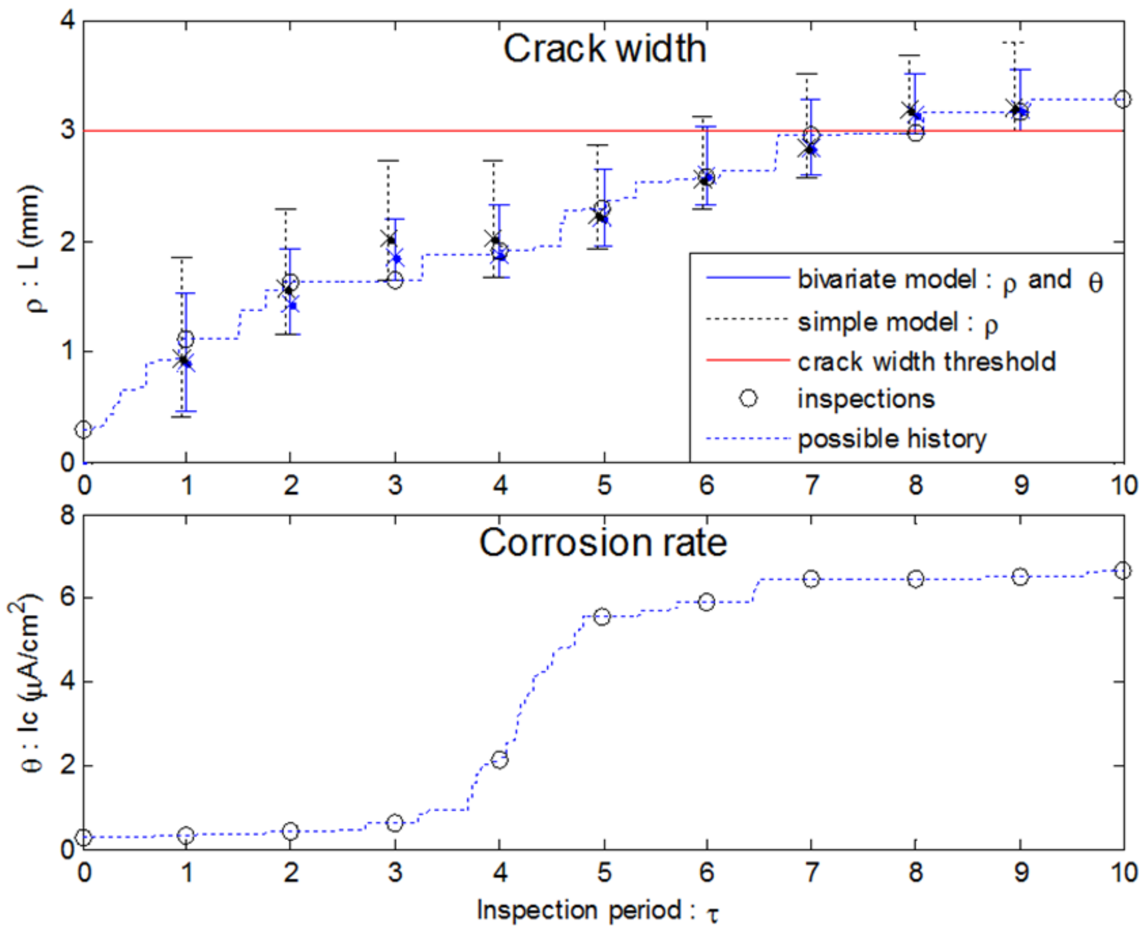

Figure 3.15 A tracking simulation of a structure and indicators on the quality of prediction of the cracking with and without taking into account the rate of corrosion.

For the durability of RC structures, Eurocode 2 expresses the failure by comparing the crack of concrete cover with a cracking threshold L. The latter depends on both the characteristic of the structure and its environmental conditions. According to Table 7.101N from Eurocode 2 (Eurocode 2005), for an exposure class XS3 (Corrosion of the reinforcement induced by chlorides from sea water) the performance of the reinforced concrete structure are assumed modified when the width of a crack is greater than or equal to 3 mm.

For each period, τ, an inspection is conducted and the observed values are noted (black circles). For each observation, the expected value of degradation is estimated for the next inspection as well as its confidence interval of 90% when taking into account all the information coming from ρ and θ (blue intervals), or just from ρ (black intervals). These quantities are obtained using stochastic simulations.

The advantage of the method is clearly illustrated in the decrease of the confidence interval. This illustration provides a better integration of the Eurocode 2 in the reliability decision of structures by adding the corrosion factor; the failure can be then defined as a level of risk related to changes in the performance of the structure for a given interval of time.

An observation that the reader might make is the existence of a lower bound on the confidence intervals at the next inspection that is in value equal to the level of degradation at the previous inspection. This observation is worthy to be commented since it is a direct consequence of using a gamma distribution for the increments. Gamma distributed increments are known to be positive, therefore, it is impossible to have negative increments that lower the lower bound of the confidence interval below the previous inspection.

3.5.2 Application to risk management

A structure is said to be safe if the probability of failure Pf at any time is lower than a given threshold. After an inspection or a level assessment, the decision criterion can be defined as the probability of having a failure before the next inspection. For the structure to be considered safe, this probability should be lower than a threshold Pf. In this section we consider a threshold $Pf = 0.05$.

The probability of a failure in the next inspection denoted $Pf(\rho_i, \theta_i)$ is a function of the current observation (ρ_i, θ_i) and given by the following equation:

$$Pf(\rho_i, \theta_i) = P(\Delta\rho + \rho_i > L | \rho = \rho_i, \theta = \theta_i) = \int_{L-\rho_i}^{+\infty} \int_{\theta_i}^{+\infty} g(x, y; \rho_i, \theta_i)\, dydx \qquad 3.10$$

For a selected range for ρ $(0 < \rho_i < L = 3\ mm)$ and θ, simulations were done for estimating the probability of failure using a Monte-Carlo method for every possible combination (ρ_i, θ_i) for a set of given parameters:
$c_1 = 2$, $c_2 = 3$, $c_3 = 0.8$, $c_4 = 0.8$, $c_5 = 0.5$, $c_6 = 0.6$, $c_7 = 1$, $\beta_\rho = 0.3$, $\beta_\theta = 0.3$.

In Figure 3.16, the state-based probability of failure curve is drawn.

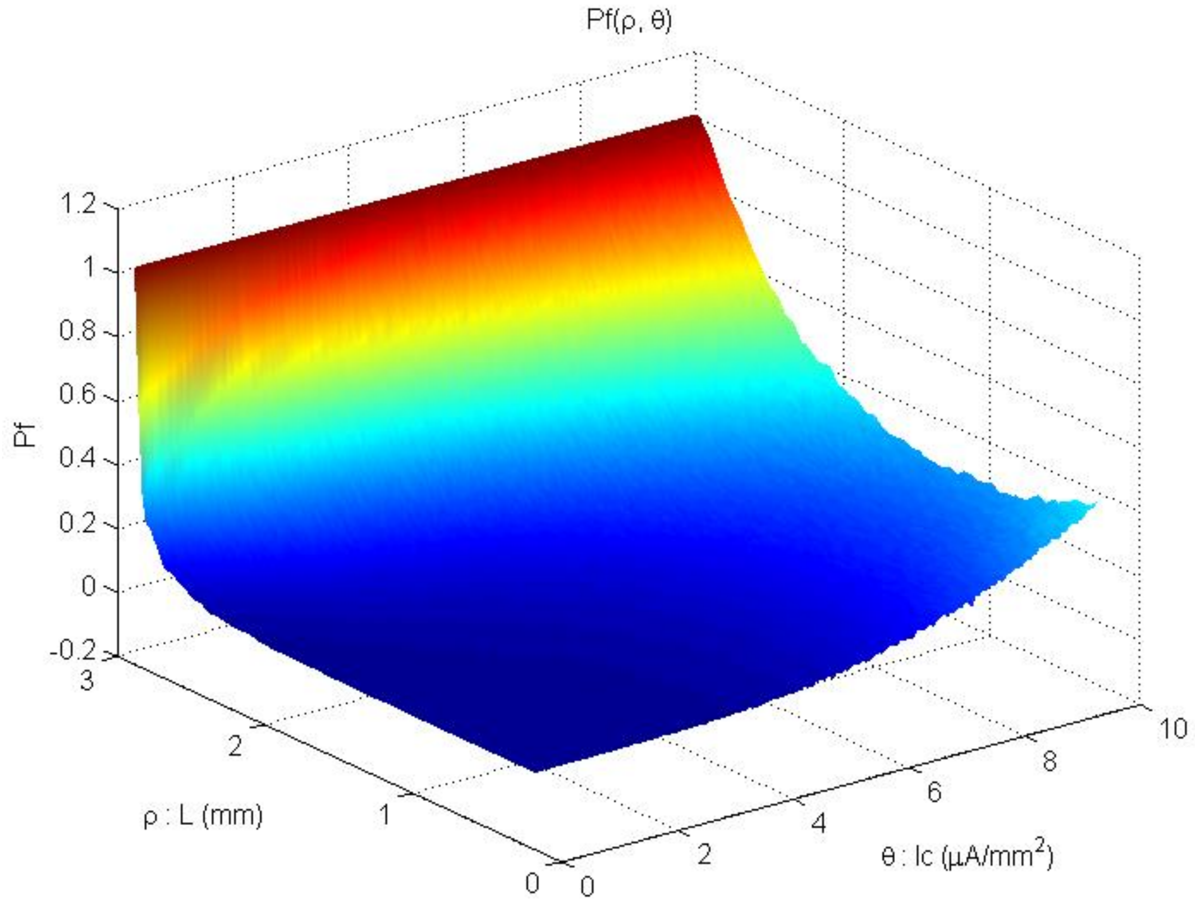

Figure 3.16 Probability of failure based on degradation level (ρ_i, θ_i)

The use of this curve in reliability-based management could be in a classical way where a Pf threshold defines an acceptance and critical areas. Therefore, we define an iso-curve as the line joining all observations (ρ_i, θ_i) having $Pf = 0.05$, and then we draw the iso-curve (green line) in Figure 3.17. The iso-curve divides the plot in two areas: an acceptance reliability area where $Pf < 0.05$, and a critical reliability area where $Pf > 0.05$.

The system is said to be safe for an observation (ρ_i, θ_i) in the acceptance reliability area (grey area) and unsafe in the critical reliability area. It is also easy to define a safety area with two thresholds and a specific attention or preventive action could be done to reduce the current risk level.

One major advantage of the proposed approach in reliability based management is that the decision can be modulated according to additional observation, given that θ can be seen as an acceleration factor of the cracking.

The aim of the following section is to investigate the effect of a potential error committed in the estimation of the parameters. Therefore, for the sake of this example, we consider a $+10\%$ error on the parameters of the bivariate process. The results are illustrated in Figure 3.17.

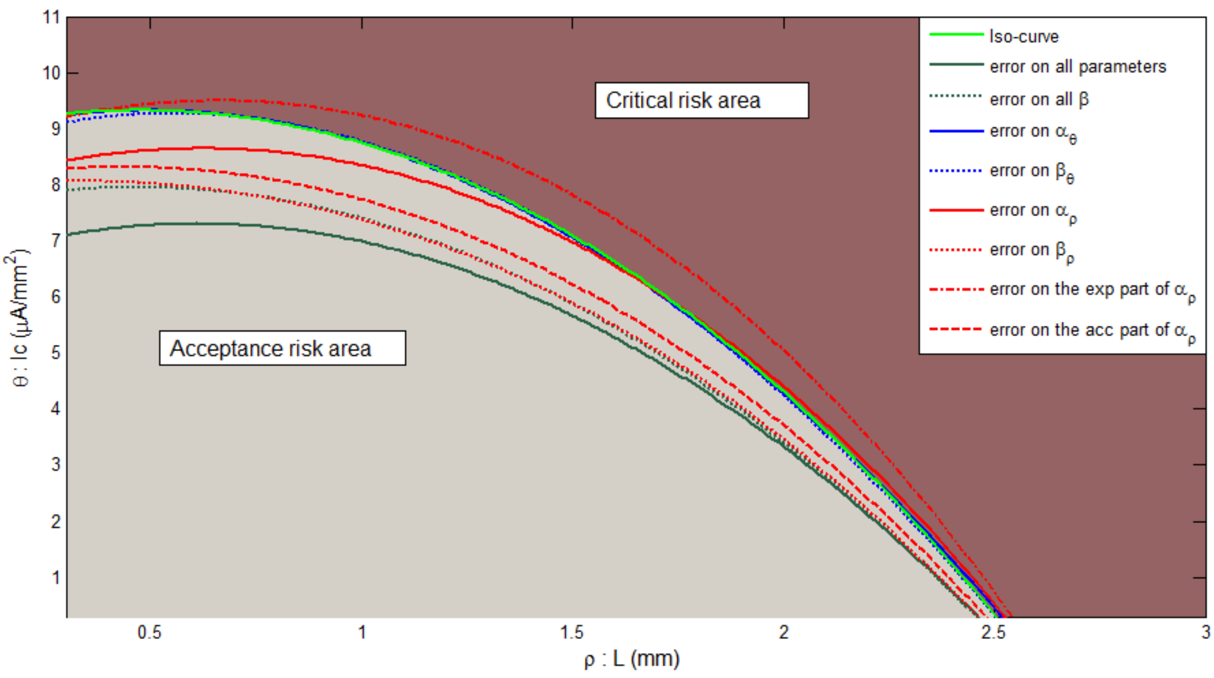

Figure 3.17 Fitted Iso-curves of the degradation levels for a 0.05 probability of failure function of a +10% error committed on several parameters of the meta-model & the simulations and mean of degradation levels

From Figure 3.17 we see that a +10% error on the parameters of the θ-process doesn't have a noticeable effect on the iso-curves, contrasting with an error committed on the parameters of the ρ-process.

An error on the exponential part (c_5) of α_ρ (Equation 3.6) pushes the iso-curve upward, opposite to the error on the acceleration part (c_6, c_7) of α_ρ (Equation 3.6) which pushes the iso-curve downwards. The parameter with the biggest impact on the iso-curve is β_ρ, followed by lesser impact from c_5, c_6 and c_7.

When 10% is added to the β_ρ parameter, the iso-curve is pushed downwards, therefore triggering an early decision generating unnecessary over costs. On the other hand, a +10% error on the exponential part of α_θ pushes the iso-curve upwards and therefore is compromising on the "safety" of the decision.

When the iso-curve moves downwards, an early decision is triggered causing additional costs. On the other hand, when the iso-curve moves upwards, the safety of the decision is compromised by a late decision. Unluckily, the decision maker cannot distinguish between these two cases. Therefore, if the decision is based on this plot, an additional safety factor needs to be considered.

We take the case of $n = 10$ structures and $T = 20$ inspections (Table 3.3). In this case we simulate 10 realizations. Then we use the MLE algorithm on these 10 realizations, resulting in 10 sets of estimated parameters for the meta-model.

The 10 corresponding iso-curves to these 10 sets are then drawn in Figure 3.18.

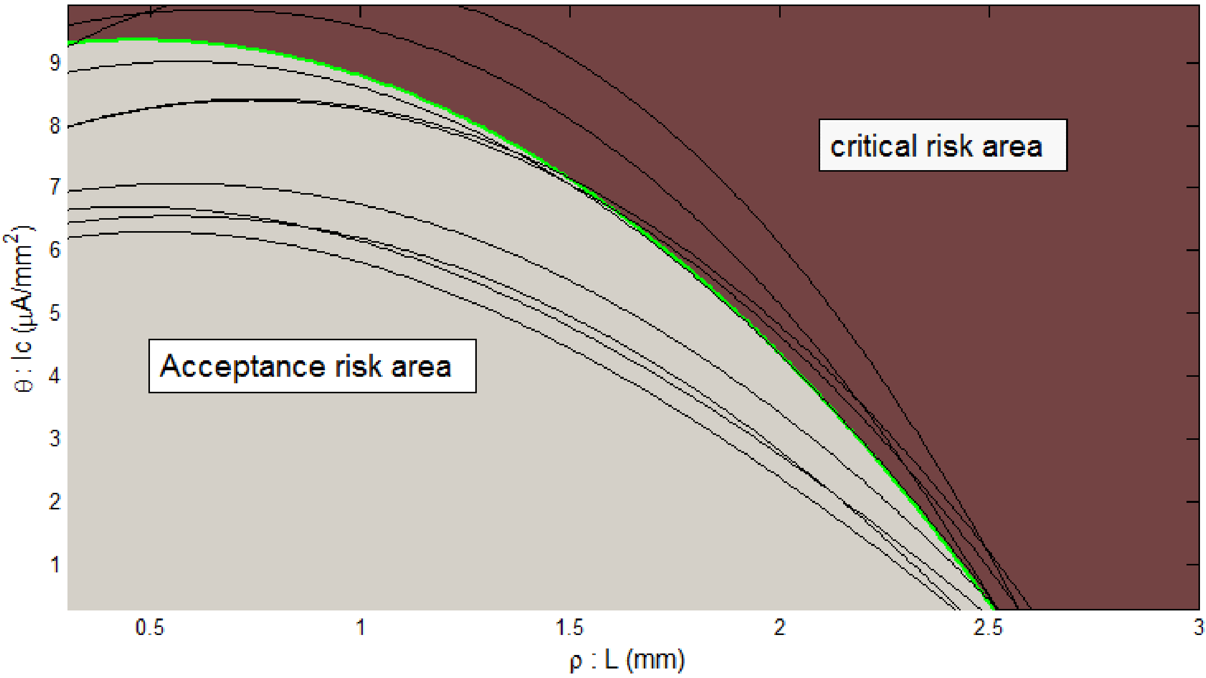

Figure 3.18 Estimated fitted iso-curves

From Figure 3.18 we witness dispersion on both sides of the green iso-curve giving us an indecisive answer on whether we are over or under the no-error iso-curve.

The safety factor is applied by translating the estimated iso-curve downwards by a distance equal to the range between the two furthest iso-curves. In Figure 3.18 the mean errors of the lowest and highest iso-curve are respectively 4% and 10% with a -17% and -4% error on the β_ρ. This safety factor will most probably generate an over cost by triggering an early action, however, secures a safer decision.

Up till now, maintenance was not taken into account in the proposed degradation model. After a maintenance action, the structure's performance is modified. Therefore, it is necessary to update the degradation model to integrate the modification, and this update is a function of the nature of the performed action. We propose in the next chapter to describe how a maintenance action can be modelled within the proposed framework.

3.6 Conclusion on probabilistic degradation modelling

The gap between sophisticated physical degradation models and the complexity of NDT results is a huge challenge in maintenance management and decision-making; the degradation model's inputs and NDT outputs are less and less related.

In this chapter, we identify principal elements of the procedure to construct a bivariate, condition-based meta-model that is based on the probabilistic pertinence and physical expertise on one hand, and on the degradation indicators directly accessible through NDT on the other hand.

The condition-based meta-model is based on data-driven, multivariate stochastic processes with gamma trends. The degradation model is based on two correlated state-dependent stochastic processes and their construction are discussed within an application of chloride induced cracking. The calibration of the processes was done via NDT data, including visual inspection or image processing ((O'Byrne et al. 2013a), to crack for instance). Expert knowledge is introduced to reflect the main useful degradation properties that the model should tackle for decision-making. This allows both to simplify the construction and the fitness of the model to the data and to overcome some limits in the current practices in civil engineering (here the correlation between the crack width and the corrosion current density).

The performance of the model was evaluated via different means, *e.g.*, adaptation to heterogeneous databases, imputing censored or truncated data, and risk management applications. The results portray the model's ability to respond to heterogeneous databases, and also remarkably improving the estimation process when taken them into account. Also, the model robustness and mathematical tractability motivate further work in risk-based decision-making.

We are aware that the construction of the model and the performance study are not based on real field data, but we hope that using this approach we can confront the problem of lack of data and that it can help in the pre-specifications of databases.

Ultimately, the aim of this work is maintenance optimization. Pathologies in civil engineering are generally multiphasic processes. Therefore, in this framework, it is important to have a degradation model that represents the whole lifetime of the deterioration process and the transitions between phases. However, in civil engineering, people tend to separate each phase alone when modelling the degradation which leads to serious consistency problems; each phase is governed by different physical phenomena (mechanical, chemical, *etc.*). Thus, different scientific communities will be treating it (*e.g.*, chloride-induced corrosion, the first phase is chemical through the diffusion of chloride, unlike the second phase which is materialistic).

In the next chapter, we illustrate the expansion of the degradation model to cover the three phases of the degradation process, and we propose an improved maintenance system with simultaneous consideration for structural performance and life-cycle cost. The meta-model construction approach will integrate the identification of different processes for each phase of the chloride induced corrosion, as well as the transitions from one phase to the other.

Chapter four

Maintenance Modelling Analysis

– Abstract –

In this chapter, we examine the third part of meta-modelling, that is, the maintenance modelling analyses. We will illustrate the modelling of degradation and maintenance for a multi-phasic degradation process. The meta-model will cover the three phases of chloride-induced corrosion for a submerged reinforced concrete structure. Maintenance actions are catalogued, an approach to model the effect of a maintenance action is proposed and illustrated, and maintenance policies are implemented and discussed to illustrate the use of the meta-model in a maintenance context.

4.1 Introduction

The purpose of this chapter is to illustrate the potential use of meta-models in a maintenance context. Therefore, we first need to address the chloride-induced corrosion by its three phases. Certainly these phases define in a classic manner the life cycle of the structure, but, as we will see, we can also allocate to each phases one or several specific maintenance operations. Thus, the problem of maintenance, in terms of identifying the actions to be implemented, joins with the field of degradation modelling problems in a risk management context.

In maintenance policies, we dispose of numerous inspection and repairing methods. Therefore, the choice of times and type of method needs to be optimized to guarantee an optimal policy. Generally speaking, an optimization processes aims to find an optimal policy within a set of constraints. Most existing maintenance systems consider only costs constraints, *i.e.*, focus on life-cycle cost minimization only. Hence, the obtained solution does not necessarily result in satisfactory long-term structural performance (Frangopol and Liu 2007).

Furthermore, as mentioned earlier, the degradation model must cover the three phases of the corrosion process. A classical approach to model degradation for mutli-phasic pathologies is to separate each phase, and deal with it using its proper laws of physics. By doing so, each phase may be treated by a separate scientific community, specialized with the physical process of the phase. For example, a chemist for the phase of diffusion of chloride, etc. As a consequence, the communication and connection between phases is not so evident making the use of these models in a dynamic maintenance system problematic. Therefore, in section 4.2.3, we develop the transitions between the phases.

In this chapter, we want to use the meta-model approach to illustrate its potential use and response to maintenance concerns. In this chapter, the meta-model will give us the ability to:

- Estimate the degradation for the three phases while fulfilling the required characteristics of a “good” degradation model defined in section 2.4.1;
- Model maintenance actions, and their effect on the degradation model;
- Estimate life-cycle costs;
- Estimate performance indexes.

In section 4.2, degradation and degradation modelling analysis are recapitulated in order to build the three phase degradation model, with a focus on modelling the transitions between phases. Section 4.2.3, maintenance analysis is discussed, where we catalogue possible maintenance actions, and model the effect of a maintenance action on the model. Also, we define the performance indexes in order to qualify the condition of structure. In Section 4.4, a decision analysis is carried out where we discuss two approaches for decision-making; a univariate approach, and a bivariate approach. Section 4.5 illustrates the use of the meta-model by considering two maintenance policies: a preventive maintenance policy, and a corrective maintenance policy to compare with. And finally, conclusions of this chapter are drawn in section 4.6.

Some of the studies in this chapter are based on the following paper: El Hajj *et al.* 2015.

4.2 Multi-phasic degradation modelling

In this chapter, we address the pathology of chloride-induced corrosion by its three phases, from chloride diffusion to failure of the structure.

As mentioned in the introduction of this chapter, the classical approach to model multi-phasic pathologies is to model each phase separately using its proper physical laws, and then define transition criteria (*e.g.*, thresholds). This approach may encounter practical difficulties. In fact, every phase is driven by a different physical process; chemical process for the diffusion phase, electro-chemical process for the corrosion phase (rust production), and electro-mechanical process for the crack propagation phase. As a consequence, the identification of physical laws and the degradation analysis for each phase may require expertise from different scientific communities; in example, chemist for the first phase, material expert for the second and third. In a dynamic context, this type of approaches can be unpractical. In fact, the communication between phases may be hindered because of the different approaches used by every community to model degradation (focusing on different outputs such as time or physical indicators). From this point of view, we see one more advantage to use a uniformed approach for degradation modelling that can be applied for the three phases here.

In this section, we aim to build the three phases degradation model. First, we recapitulate quickly the analysis done in section 3.2 where we discussed the selection of indicators for the three phases and the transitions between phases. Second, the mathematical formulation of the phases is detailed using the same approach introduced and defined in subsection 3.3.1. Finally, in subsection 4.2.3, the transitions between phases are modelled.

4.2.1 Degradation analysis

The degradation analysis of chloride-induced corrosion was carried out in details in section 3.2. In this subsection, we focus on the main conclusions.

Corrosion of reinforcement steel is known to be one of the major causes of degradation of reinforced concrete (RC) structures (Bastidas-Arteaga and Schoefs 2012). Its life cycle can be discretised into three phases (§ 3.2):

- Diffusion phase: controlled by the diffusion of chlorides into concrete;
- Corrosion phase: dominated by the chemical reaction of corrosion generating corrosion products that build internal stress on the concrete;
- Deterioration phase: controlled by the propagation of the crack until failure.

As seen in section 3.2.1, the proposed probabilistic degradation model is based on a small number of "physical" indicators that reflect the level of degradation throughout the life cycle of the pathology. Two indicators are chosen per phase, ρ_i and θ_i, indicators for the condition and potential of evolution respectively, where i stands for the number of the phase. The choice of an indicator were based on its accessibility through NDT inspections, and on its weight on the assessment of the degradation level.

To resume, the following physical indicators were chosen:

- For the 1st phase: Chloride concentration at cover depth $[Cl^{-}]$, and concrete's pH.
- For the 2nd phase: Internal tensile stress, and corrosion current density.
- For the 3rd phase: Crack width, and corrosion current density.

4.2.2 Modelling analysis

In section 3.3.1, we defined the general degradation modelling approach using a bivariate process written $(\rho_{i,t}, \theta_{i,t})_{\forall t\geq 0}$, where:

i. $(\rho_{i,t})_{\forall t\geq 0}$ describing a condition state modelled as a SDGP
ii. $(\theta_{i,t})_{\forall t\geq 0}$ describing a potential of evolution modelled as a SDGP
iii. i is the number of phase, $i \in [1,2,3]$

In this section, we formulate the degradation model for each phase using this approach.

This approach was detailed for the third phase of degradation in subsection 3.3.2. Therefore, we start by reminding the reader by the third phase's functions, and then we proceed to formulate for the second and first phases.

Third phase (c.f. 3.3.2)

From subsection 3.2.1.3, the crack propagation phase is characterized by:

- $(\rho_{3,t})_{\forall t\geq 0}$, represents the width of the crack *a* (mm).
- $(\theta_{3,t})_{\forall t\geq 0}$, models the corrosion current density i_{corr} (µA/cm2).

This model is sequential in the sense that in a first step we seek to characterize the evolution in terms of changes in the corrosion current density (θ_3, Equation 3.3) before doing so for the cracking itself (ρ_3, Equation 3.4).

The distributions of the increments were as follow, $\forall(\rho_3, \theta_3) > 0$:

$$\Delta\theta_3(\tau_3\,;\,\rho_3,\theta_3) \sim gamma\left(y{:}\,\alpha_{\theta_3}(\rho_3,\theta_3).\tau_3\,,\beta_{\theta_3}\right) \qquad (c.f.\ 3.3)$$

$$\Delta\rho_3(\tau_3\,;\,\rho_3,\theta_3,\Delta\theta_3) \sim gamma\left(x{:}\,\alpha_{\rho_3}(\rho_3,\theta_3,\Delta\theta_3).\tau_3\,,\beta_{\rho_3}\right) \qquad (c.f.\ 3.4)$$

With the suitable shape functions:

$$\alpha_{\theta_3}(\rho_3,\theta_3) = (c_3.\rho_3 + c_4).e^{\frac{-(\theta_3-c_1)^2}{c_2}} \qquad (c.f.\ 3.5)$$

$$\alpha_{\rho_3}(\rho_3,\theta_3,\Delta\theta_3) = \left(c_6.\left(\theta_3 + \frac{\Delta\theta_3}{2}\right) + c_7\right).e^{-c_5.\rho_3} \qquad (c.f.\ 3.6)$$

Second phase

From subsection 3.2.1.2, the crack initiation phase is characterized by two parameters:

- $(\rho_{2,t})_{\forall t\geq 0}$, represents the internal tensile stress (MPa).
- $(\theta_{2,t})_{\forall t\geq 0}$, models the corrosion current density « i_{corr}» $(\mu A/cm^2)$.

The simulation of the phase is sequential. Using the same *cause-effect* analyse used for the third phase (*c.f.* 3.3.1), here, in a first step we characterize the evolution in terms of changes in the corrosion current density (θ_2) before doing so for the stress (ρ_2). Furthermore, in Figure 3.4 we see that the ρ_2 process has an S-shaped tendency, and θ_2 process has an L-shaped tendency.

Therefore, we propose $\forall(\rho_2, \theta_2) > 0$:

$$\Delta\theta_2(\tau_2\ ;\ \rho_2, \theta_2) \sim -\ gamma\big(y: \alpha_{\theta_2}(\ \rho_2, \theta_2).\tau_2\ , \beta_{\theta_2}\big) \qquad 4.1$$

$$\Delta\rho_2(\tau_2\ ;\ \rho_2, \theta_2, \Delta\theta_2) \sim gamma\big(x: \alpha_{\rho_2}(\rho_2, \theta_2, \Delta\theta_2).\tau_2\ , \beta_{\rho_2}\big) \qquad 4.2$$

In Eq. 4.1, we notice the $(-)$ sign in the equation, that is to respect the monotonically decreasing tendency of the corrosion current density (Figure 3.2, Figure 3.4), represented by θ_2. Also, the $gamma\big(y: \alpha_{\theta_2}(\ \rho_2, \theta_2).\tau_2\ , \beta_{\theta_2}\big)$ function is the truncated gamma probability density function ensuring a non-negative i_{corr} by truncating all possible increments that are bigger that θ_2.

The appropriate shape-functions for the S-shaped ρ_2 and the L-shaped θ_2 are:

$$\alpha_{\rho_2}(\rho_2, \theta_2) = (b_3.\rho_2 + b_4).e^{\frac{-(\theta_2 - b_1)^2}{b_2}} \qquad 4.3$$

$$\alpha_{\theta_2}(\rho_2, \theta_2, \Delta\theta_2) = \left(b_6.\left(\theta_2 + \frac{\Delta\theta_2}{2}\right) + b_7\right).e^{-b_5.\rho_2} \qquad 4.4$$

First phase

From subsection 3.2.1.1, the corrosion initiation phase is characterized by two parameters:

- $(\rho_{1,t})_{\forall t\geq 0}$, represents the concentration of chloride at the surface of the steel $[Cl^-]$.
- $(\theta_{1,t})_{\forall t\geq 0}$, models the basicity of the concrete pH.

For this phase, the *cause-effect* analyse dictates that in a first step we characterize the evolution in terms of changes in $[Cl^-]$ (ρ_1) before doing so for the pH (θ_1). In fact, the diffusion of chlorides in the concrete matrix is the cause of decreasing of pH.

In Figure 3.4, we see that the ρ_1 process has an S-shaped tendency, and θ_1 process has an L-shaped tendency.

As a result, we propose $\forall(\rho_1, \theta_1) > 0$:

$$\Delta\rho_1(\tau_1;\ \rho_1, \theta_1) \sim gamma\left(x{:}\ \alpha_{\rho_1}(\ \rho_1, \theta_1).\tau_1, \beta_{\rho_1}\right) \quad 4.5$$

$$\Delta\theta_1(\tau_1;\ \rho_1, \theta_1, \Delta\rho_1) \sim -\ gamma\left(y{:}\ \alpha_{\theta_1}(\rho_1, \theta_1, \Delta\rho_1).\tau_1, \beta_{\theta_1}\right) \quad 4.6$$

In Eq. 4.6, we notice a $(-)$ sign in the equation; In fact, the concrete's pH, represented here by θ_1, has a monotonically decreasing evolution (Figure 3.4). Furthermore, the $gamma\left(x{:}\ \alpha_{\theta_1}(\rho_1, \theta_1, \Delta\rho_1).\tau_1, \beta_{\theta_1}\right)$ function is a truncated gamma probability density function ensuring a non-negative pH by truncating all possible increments that are bigger that θ_1.

The appropriate shape-functions for the S-shaped ρ_1 and the L-shaped θ_1 are:

$$\alpha_{\rho_1}(\rho_1, \theta_1) = (a_3.\theta_1 + a_4).e^{\frac{-(\rho_1 - a_1)^2}{a_2}} \quad 4.7$$

$$\alpha_{\theta_1}(\rho_1, \theta_1, \Delta\rho_1) = \left(a_6.\left(\rho_1 + \frac{\Delta\rho_1}{2}\right) + a_7\right).e^{-a_5.\theta_1} \quad 4.8$$

4.2.3 Modelling the transitions

The transition between phases is modelled by defining thresholds on the condition level indicators:

First transition, from the first phase to the second phase, is governed by the chloride concentration at the surface of the steel (Angst et al. 2009). Once the $[Cl^-]$ reaches or exceeds a threshold $[Cl^-]_{thr}$, the corrosion initiates and a transition to the second phase occurred. Angst *et al.* (2009) have shown that the value of the threshold $[Cl^-]_{thr}$ is heavily discussed in the literature and many values are proposed. As a consequence, its value will be different for each simulation of the process, and is modelled using a uniform distribution on a pre-defined interval [0.4% – 0.5%]. These values are inspired from physical expertise.

The second transition, from the second phase to the third phase, is governed by the internal stress on the concrete. In fact, the crack starts (*i.e.*, the third phase starts) when the internal stress exceeds the ultimate tensile strength of the concrete σ'_t. Because of material impurities, we will consider that the value of σ'_t follows a uniform distribution on the interval $[\sigma'_t - 0.2\ , \sigma'_t + 0.2]$. In the Eurocode, the value of σ'_t is calculated from the value of the ultimate compressive strength, σ'_c. Here we consider a $\sigma'_c = 30\ MPa$, according to section 3.1.6. of the Eurocode (Eurocode 2005), σ'_t is equal to 2.9 MPa. Therefore, the value of σ'_t will follow a uniform distribution in the interval $[2.7\ MPa\ -\ 3.1\ MPa]$.

The third transition, or failure, is defined when the crack width exceeds a code-defined maximum crack width, w_{lim}. Unlike the first two transitions, w_{lim} is modelled as a determinist value in the model. In fact, according to Table 7.101N from Eurocode 2, for an exposure class XS3 (Corrosion of the reinforcement induced by chlorides from sea water) the performance of the reinforced concrete structure are assumed modified when the width of a crack is greater than or equal to $w_{lim} = 3\ mm$ (Eurocode 2005).

4.3 Maintenance analysis

In maintenance analysis, we should focus to specify the needs in terms of modelling the maintenance and the decision criterion. The identification and the conditions of implementation of maintenance actions allow to see the relevance of the modelling of the degradation process in three phases. Also, an important point task is to model the effect of a maintenance action on the degradation.

In this section, first, possible maintenance actions are catalogued, then, a proposed technique to model the effect of maintenance actions on the model is explained, and finally, a performance index is defined.

4.3.1 Repairing actions catalogue

Firstly, we look into possible repairing actions that can be applied in the case of chloride-induced corrosion. For this pathology, we have numerous physics of degradation; *e.g.*, chemical such as diffusion of chlorides, electrical such as corrosion, and physical such as crack propagation. Therefore, we have a wide range of repairing actions that can be performed in order to re-establish the destined performance of the structure, or to extend its life time.

A great effort has been made to organize the protection and reparation of reinforced concrete structure. A European approach to this mission is based on the establishment of performance requirements for the repair process, which are evaluated according to recommended conformity tests. This approach is known as the Performance Based Approach (PBA), and is embraced by the European Standard EN 1504. The approach is based on 11 principles that can be used to prevent or stabilize the degradation processes. The principles differ from each other by the type of damage and its cause (*e.g.*, chemical, electrochemical, physical) for the repair defined according to the type of damage and their causes (chemical, electrochemical or physical).

When choosing a suitable repair method, the choice is based on an analysis of the principles which best satisfy them. Principles 1 to 6 and their correlated methods are related to defects in concrete, while principles 7 to 11 are related to defects due to corrosion of the reinforcement.

In Table 4.1, we summarize the principles as well as examples of reparation methods for each one.

The list of methods is bigger than the one presented in this table, but here, we will limit the list to the most common ones. The reader may refer to EN 1504 for additional details and methods. A technical guide on maintenance methods can be found on http://durati.lnec.pt, however, here we give a general description of common repair methods: Concrete injection, concrete replacement, chloride extraction and cathodic protection.

Table 4.1 Principles and example of reparation methods according to EN 1504

Principle		Definition	Examples of methods
Principles related to defects in concrete			
1	PI	Protection against ingress	Hydrophobic impregnation, filling cracks
2	MC	Moisture control	Electrochemical treatment, hydrophobic impregnation
3	CR	Concrete restoration	Replacing elements, recasting with concrete
4	SS	Structural strengthening	Adding or replacing embedded or external reinforcing bars, injecting cracks
5	PR	Increasing physical strength	Overlaying or coating
6	RC	Resistance to chemicals	Impregnation
Principles related to reinforcement corrosion			
7	RP	Preserving or restoring passivity	Increasing cover with additional mortar or concrete, electrochemical chloride extraction
8	IR	Increasing resistivity	Hydrophobic impregnation, coating
9	CC	Cathodic control	Limiting oxygen content (at the cathode) by saturation or surface coating
10	CP	Cathodic protection	Applying an electrical potential
11	CA	Control of anodic areas	Applying corrosion inhibitors to the concrete, coating

According the EN 1504-9 the methodology for selecting the protection or repair methods should be:

- to adopt the principle adequate to accomplish the intervention option taken;
- to choose an appropriate method of repair that satisfies the principle adopted;
- to choose the products or systems that comply with EN 1504;
- to define the requirements for quality control during repair works and for future inspection and maintenance.

4.3.1.1 Hydrophobic impregnation (HI)

These processes are applied to the concrete surface to prevent the ingress of water and aggressive substances associated with various degradation processes, in this case chlorides. The treatment by hydrophobic impregnation involves the application of low viscosity liquid products such as silanes and siloxanes, creating a water-repellent surface that prevent the ingress of liquids that occurs by capillary action. The appearance of the concrete surface does not change.

Treatments can be applied to new structures, *i.e.*, before the initiation of the deterioration processes, or included as part of a repair system. If the deterioration process has already induced damage in concrete, the application of this treatment requires previous repair by concrete replacement. The treatment is not effective where deterioration has already taken place.

4.3.1.2 Chloride extraction (CE)

Chloride extraction (CE), sometimes called desalination, is an electrochemical process to remove chloride ions from a chloride contaminated concrete through ion migration. An anode embedded in an electrolyte medium is temporarily applied on the surface of the concrete, forcing the Cl^- to migrate outside the concrete.

4.3.1.3 Concrete replacement (CR)

Concrete replacement is used for restoring the original load-carrying capacity of damaged concrete or replacing a highly contaminated concrete.

In the case of chloride-induced corrosion, the concrete replacement can be applied at three different levels, generally respectively to the three phases:

i. CR level 1 [*CR1*]: A preventive repair strategy in which the structure is repaired before corrosion initiation. Chloride-contaminated concrete cover is repaired by replacing few centimetres of material for slabs and beams (before concrete cover depth). Corroded bars are not replaced;
ii. CR level 2 [*CR2*]: A corrective repair strategy in which repair takes place after corrosion initiation, however, the loss of cross-sectional area of rebars is not significant;
iii. CR level 3 [*CR3*]: A corrective repair strategy in which repair takes place after severe concrete cracking where the loss of cross-sectional area of rebars are significant.

For [*CR2*] and [*CR3*], cracked/chloride-contaminated concrete cover is repaired by removing about 6 cm of material for slabs and beams. Corroded bars are replaced.

4.3.1.4 Cathodic protection (CP)

Cathodic protection (CP) is an electrochemical technique used to control the corrosion by making it the cathode of an electrochemical cell. CP systems protect metal reinforcement bars in concrete buildings and structures from corrosion, and, in some cases, can prevent stress corrosion cracking. It prevents corrosion by converting the active anodic sites on the reinforcement surface to passive cathodic sites by supplying electrical current or free electrons from an alternate source.

CP may be achieved by two ways depending on the supplied source of power: by the use of an impressed DC current from an electrical source, or by the use galvanic action (also known as sacrificial anodes).

Galvanic action *(or sacrificial anode)*

In the application of passive cathodic protection, galvanic (or sacrificial anode) is selected. The sacrificial anode is more electrochemically active (lower electrode potential) than the corroded reinforcement (cathode) and is electrically connected to the surface of the steel where it is exposed to an electrolyte.

The potential of the steel surface is then polarized until the surface has a uniform potential. At that stage, we are protecting the cathode. The corrosion is transferred from the reinforcement steel to the sacrificial anode, consuming material until eventually replaced.

The polarization of the reinforcement steel is done through migration of electrons from the anode to the cathode. Therefore, it is important that these two metals have a good electrically conductive contact.

This type of CP is mostly used for local protection where we have a clear idea where the steel is under corrosion reaction.

Impressed current systems *(ICS)*

ICS is generally an option where galvanic anodes fail economically or physically to deliver enough current in order to provide protection, for example larger structures and higher electrolyte resistivity.

ICS is a set of anodes connected to a direct current (DC) power source. Sometimes the DC is supplied by means of a transformer-rectifier connected to an alternative current (AC) powered by a supply, solar panels, wind power or gas powered thermoelectric generators.

Then, the DC negative pole is connected to the reinforcement steel to be protected by ICS, and the positive is connected to the anodes. The output of the ICS is adjusted in a way to provide sufficient current to provide cathodic protection.

4.3.2 How to integrate the effect of a maintenance action in the degradation meta-model

A very important feature to have in every maintenance model is the ability to model the effect of a maintenance action (*e.g.*, the ability to model imperfect maintenance actions that does not restore the condition of the system to as good as new). Therefore, as a continuation to our meta-modelling approach, here we suggest discussing how and what should be integrated in the degradation model to give us the ability to alter the main tendencies of the degradation after a maintenance action.

A maintenance action can have two different types of effects on the degradation process:

i. On the future evolution of the degradation process.
ii. On the current level of degradation.

In i), a maintenance action modifies the rate of degradation, in example a CP decelerates the process, and a CR accelerates it (*e.g.*, after CR, a small chloride content remains in the unremoved concrete inducing a chloride diffusion from the old material that can accelerate corrosion initiation).

Moreover, in ii), an action can change the level of degradation, and here we define an after-maintenance level of degradation (*e.g.*, CE removes given quantity of chlorides inside the concrete but not all of it).

In Table 4.2, the effects of the maintenance actions on the degradation are summarized.

Table 4.2 Maintenance action effect on the meta-model

Maintenance actions	*1st Phase*		*2nd Phase*		*3rd Phase*	
	ρ_1	θ_1	ρ_2	θ_2	ρ_3	θ_3
CE	$a*$	$a*$	*NA*	*NA*	*NA*	*NA*
HI	*d*	*d*	*d*	*d*	*NA*	*NA*
CR1	$a*$	$a*$	*NA*	*NA*	*NA*	*NA*
CR2	$a*$	$a*$	$a*$	$a*$	*NA*	*NA*
CR3	$a*$	$a*$	$a*$	$a*$	$a*$	$a*$
CP	*d*	*d*	*d*	*d*	*d*	*d*

In Table 4.2, the effect of a maintenance action is represented using letters and symbols as follows:

- *d*: decelerates the degradation process.
- *a*: accelerates the degradation process.
- *a**: accelerates the process and lower the degradation level.
- *NA*: Not applicable alone for phase.

4.3.2.1 Effect on the future evolution

The shape functions govern the tendencies of the degradation process, therefore, for example if the shape function is lowered, in average the increments will be smaller. It is as if the degradation process is being decelerated.

Hence, when it comes to the speed and evolution of the degradation, we propose to model the effect of a maintenance action on the processes directly through the shape functions by introducing two new parameters, m_1 and m_2, which can be defined respectively as the degradation acceleration factor and the effect of unremoved concrete after maintenance factor.

In the proposed effect modelling, we consider that a maintenance action has none on the scale parameter β.

In Figure 4.1, the modification of the shape function is illustrated for the S-shaped and L-shaped tendencies.

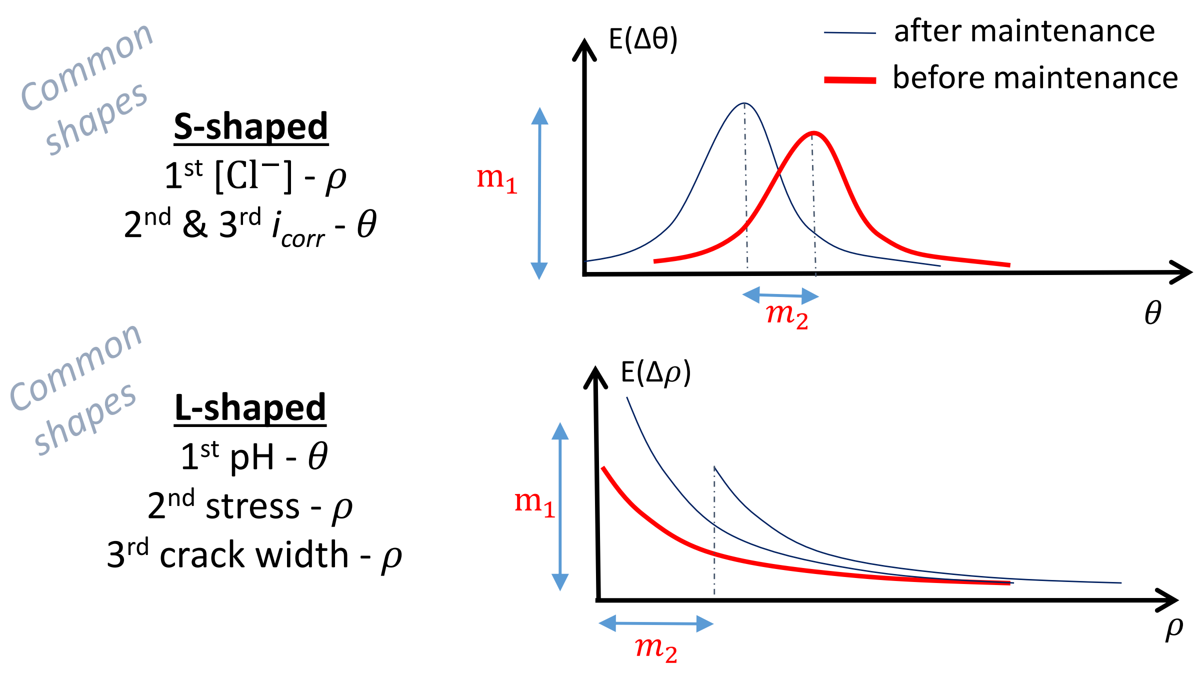

Figure 4.1 Mathematical model of a maintenance action on the shape functions

From Figure 4.1, we can see the illustrated effect of the maintenance parameters on the shape functions. For instance, for the S-shaped tendency represented by a bell-shaped shape function, using m_2, the shape function is translated to the left side, as a result, the inflection point of the S-shape is brought earlier. Moreover, the shape function is raised using m_1, as a result, increments will increase in average, generating a faster degradation. A similar reasoning can be carried out for the L-shaped function.

The first maintenance parameter is introduced by multiplying the shape function by a constant m_1 appropriate to an action. The effects of m_1 are the following, if:

- $m_1 > 1$, the process is accelerated.
- $m_1 = 1$, the maintenance action has no effect on the evolution of the process.
- $m_1 < 1$, the process is decelerated.

The second maintenance parameter m_2, is introduced as a translation parameter in the equation of the shape functions.

To conclude, for an "S-shaped" trend, the effect of a maintenance action on its shape function is modelled as follows:

$$\alpha_S(x,y) = \boldsymbol{m_1} \times g(x,y) \times e^{\frac{-((x-\boldsymbol{m_2})-a_1)^2}{a_2}} \qquad 4.9$$

Similarly, for an "L-shaped" trend, the effect of a maintenance action on its shape function is modelled as follows:

$$\alpha_L(x,y) = \boldsymbol{m_1} \times g(x,y) \times e^{-a_3.(x-\boldsymbol{m_2})} \qquad 4.10$$

By modelling the effect of maintenance using self-explanatory parameters, we benefit greatly on two stakes:

- In identifying m_1 and m_2 we can give a physical meaning to the effect of a maintenance action in terms of translation and rate of degradation.
- It is easier to compare the effect of different maintenance actions using parameters that physically quantify a maintenance action.

Maintenance and repairing techniques are progressively being studied, and their effects on the physical process are increasingly being examined. Therefore, the last remaining part in modelling the maintenance effect is to quantify m_1 and m_2. The estimation process of these parameters are beyond the scope of this section, but an MLE procedure similar to the one used in section 3.3.3 can be used, or a more relevant approach based on an expertise-based Bayesian estimation.

4.3.2.2 Effect on the current level

On the other hand, a maintenance action can also modify the level of degradation. The state-dependent meta-model benefits from the markovian property; as a consequence, the effect on the level of degradation is modelled by giving an appropriate after-maintenance value for the degradation level.

In example, after a chloride extraction action, the concentration of chlorides is reduced to few traces in the concrete. In the SDGP, all it takes is to give a level to the after maintenance $[Cl^-]$. However, a value needs to be chosen.

Clearly, there is a high level of uncertainty on these values, and there effect has a big impact on the future prognosis. Therefore, a proposed approach to choose the after-maintenance values is that they follow a uniformly distributed interval determined from expert judgment.

4.3.2.3 Illustration of the effect using a Cathodic Protection (CP)

In order to illustrate the effect of a maintenance action, in this example, we consider the application of the two types of CP: galvanic and impressed current. The two actions are applied at the start of the third phase of the chloride-induced corrosion (*i.e.*, crack propagation).

From Table 4.2, we see the effect of a CP on the speed of evolution on of the corrosion process: a CP decelerates the corrosion. Using the approach, introduced in the previous section, we model the effect of a CP. Hence, the appropriate shape functions are:

$$\alpha_{\theta_3}(\rho_3, \theta_3) = m_1(c_3.\rho_3 + c_4).e^{\frac{-((\theta_3 - m_2) - c_1)^2}{c_2}} \qquad 4.11$$

$$\alpha_{\rho_3}(\rho_3, \theta_3, \Delta\theta_3) = k_1 \times \left(c_6.\left(\theta_3 + \frac{\Delta\theta_3}{2}\right) + c_7\right).e^{-c_5.(\rho_3 - k_2)} \qquad 4.12$$

Where $[m_1, m_2, k_1, k_2]$ are the maintenance parameters.

Let's consider, for the sake of this example, that a galvanic CP will slows the corrosion process by 10%, and the impressed current CP slows by 20% (these values are not based on real data but we can consider that their respective ranges are realistic).

The corresponding maintenance parameters are:

- For the galvanic CP: $[m_1, m_2, k_1, k_2] = [0.9, 0, 0, 0]$.
- For the impressed current CP: $[m_1, m_2, k_1, k_2] = [0.8, 0, 0, 0]$.

In Figure 4.2, the mean simulations of the bivariate process after consideration of both CP actions are illustrated.

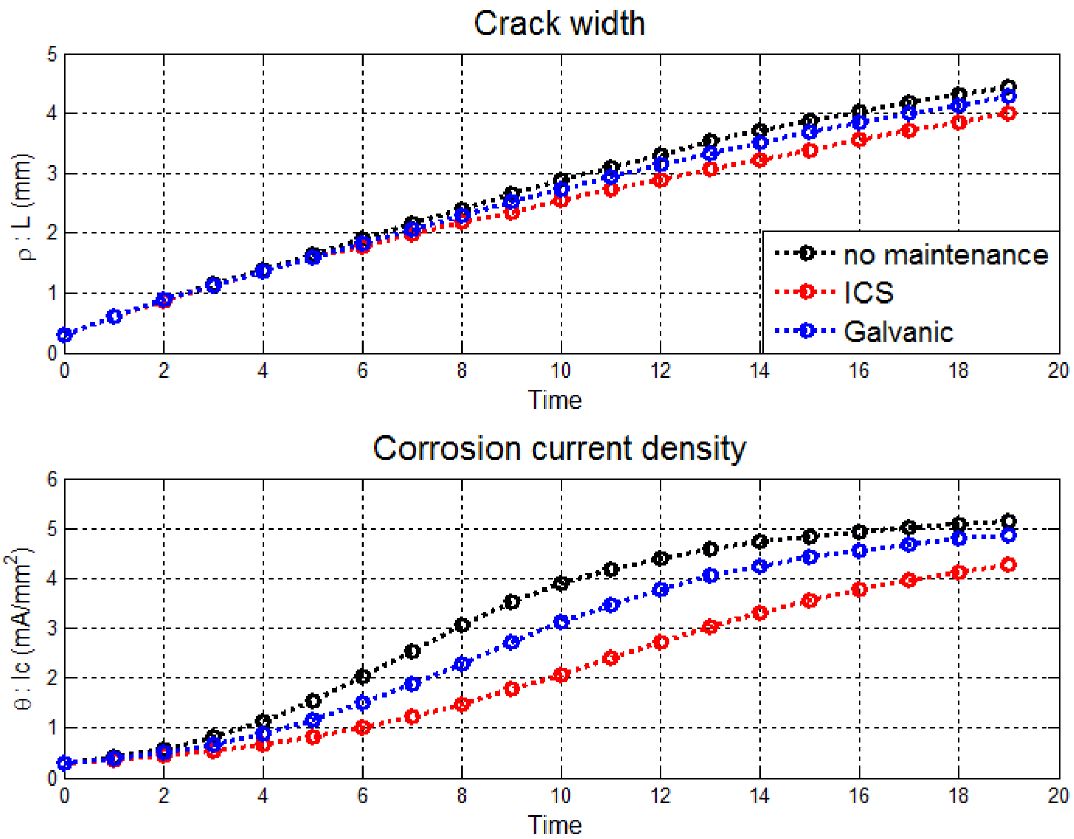

Figure 4.2 Average simulations in case of two types of [CP]

4.3.2.4 Conclusion on the maintenance effect modelling

As a conclusion, a maintenance action is modelled by introducing two new parameters into the shape functions of each phase. That is a total of six maintenance parameters that calibrate the effect of a maintenance action, instead of updating the original 27 parameters of the model.

4.3.3 Performance indexes

Maintenance management decisions are based on the performance or condition of the structure. Typically maintenance interventions should be prioritized to structures with unacceptable performance levels. In order to describe the performance of a structure, appropriate performance indicators are needed. In the literature we have many types of performance indicators, the most common ones are (Frangopol *et al.*, 2007):

Condition indexes

Condition indexes are based on the value of the inspection (Gattulli and Chiaramonte 2005). All the possible states that the structure can have are discretised into classes, then, the value of a measurement determine to which class it belongs, hence, determine the structure's condition. This type of approaches to performance indexing is traditional, yet remains practical and widely used, *e.g.*, visual inspection condition rating index is traditionally used to measure the bridge's remaining load-carrying capacity (Pontis 2001).

Reliability indexes

The reliability index is used to quantify the structure safety level and can thus be used as a performance indicator (Nowak and Collins 2012; Lee, Yang, and Ruy 2002).

Safety indexes

For these indexes, the structure can be safe or unsafe, *e.g.*, in the UK regulations, the safety index is defined as the ratio of available to required live load capacity (DB12/01 2001), and the structure performance is considered unacceptable if the value of safety index drops below 0.91.

In this study, we consider a condition indexing approach (CI). We classify the structure using 10 states or classes. Each phase is divided into 3 classes, starting from a $CI = 9$ and going down to $CI = 0$. A $CI = 9$ is associated with a low concentration in chlorides (early phase of chlorides diffusion), and a $CI = 0$ associated with a failure.

The number and lengths of the CI classes are the decision variables that must be optimized within the maintenance criteria. An optimization procedure could be on the analysis of the MDP by relying on classical results from the research for an optimal decision structures and on associated algorithms related in this dynamic programming context (Policy Iteration Algorithm for example, (Littman, Cassandra, and Kaelbling 1995; Littman 1996)). However, here, we are more interested in the validation of the global approach, than the construction of the optimization procedure.

Now, it is possible to define classes based on two inspections (ρ_i and θ_i) or on one. Since thresholds to end phases are defined on ρ_i, we define the CI classes based on the inspected value of ρ_i. Nonetheless, an example of a bivariate CI classing is illustrated briefly in subsection 4.4.3.

CI classes are defined, within each phase, by discretising the region between the horizontal axis and the threshold line of ρ_i into three un-even regions.

We define the discretization of the CI classes using square root intervals that transforms three even intervals into three un-even intervals. The closer the interval to the threshold, the narrower is. In order to define a unique approach for the discretization each phase, we create a scale axis that will determine the three intervals by applying the scale between the threshold of the phase and the horizontal axis.

In Figure 4.3, the square root intervals discretization is illustrated.

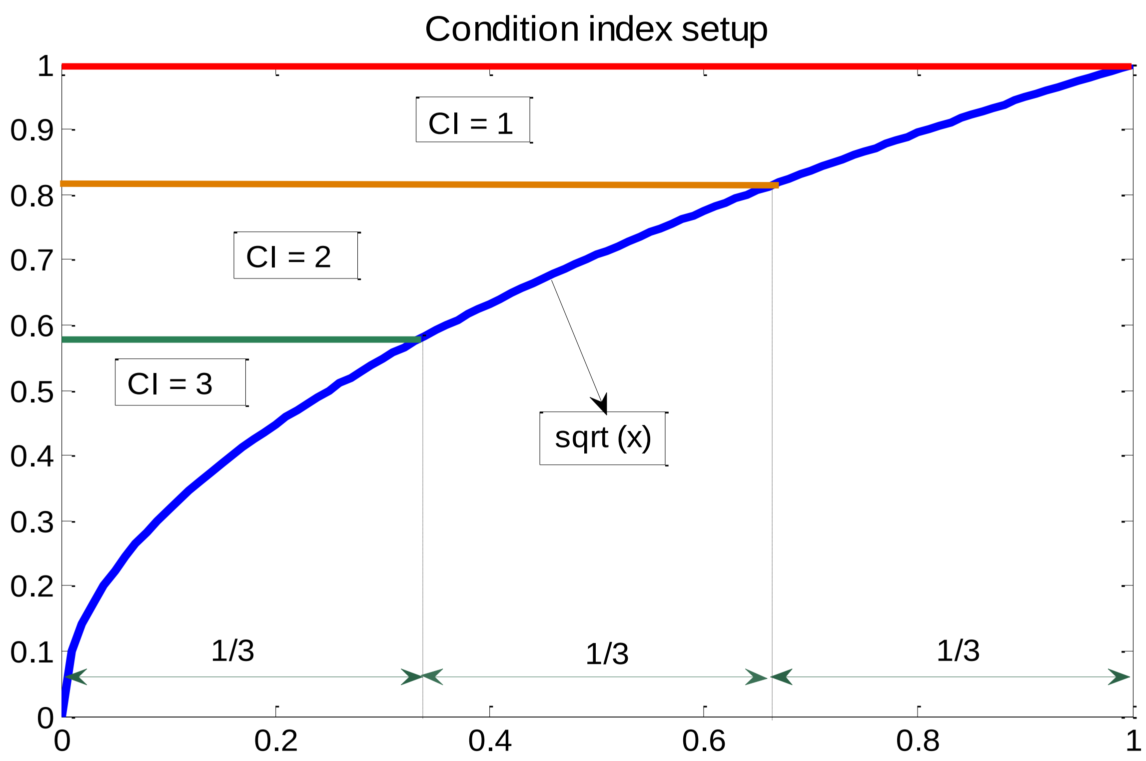

Figure 4.3 Condition states setup

In this figure, the horizontal axis extends from [0 1], and is discretized into three even intervals, each spanning over $1/3$ unit. Then, through the square root function, we transform the three even intervals, into three un-even intervals on the y axis. As a result of this transformation, the discretised y axis, also extends on [0 1] since $\sqrt{[0\ 1]} = [0\ 1]$, and form the scale that is used to define the condition indexes on each phase.

To better illustrate the classification, in Table 4.3 the CI classes are summarized.

Using this approach to condition indexes generates two practical characteristics:

- The ability to quantify the condition of the degradation on 10 levels scale; 3 per phase, and 1 for failure.
- The ability to be more certain when the degradation level is near a threshold since CIs classes are modelled to become narrower as we come nearer to the threshold.

Table 4.3 Ranges of the condition indexes

1st phase		*2nd phase*		*3rd phase*	
CI	*Range*	*CI*	*Range (MPa)*	*CI*	*Range (mm)*
9	$0 \le \frac{\rho_1}{[Cl^-]_{th}} < \sqrt{\frac{1}{3}}$	*6*	$0 \le \frac{\rho_2}{\sigma_t'} < \sqrt{\frac{1}{3}}$	*3*	$0 \le \rho_3 < 1.65$
8	$\sqrt{\frac{1}{3}} \le \frac{\rho_1}{[Cl^-]_{th}} < \sqrt{\frac{2}{3}}$	*5*	$\sqrt{\frac{1}{3}} \le \frac{\rho_2}{\sigma_t'} < \sqrt{\frac{2}{3}}$	*2*	$1.65 \le \rho_3 < 2.32$
7	$\sqrt{\frac{2}{3}} \le \frac{\rho_1}{[Cl^-]_{th}} < 1$	*4*	$\sqrt{\frac{2}{3}} \le \frac{\rho_2}{\sigma_t'} < 1$	*1*	$2.32 \le \rho_3 < 3$
				0	$\rho_3 \ \ge 3 = w_{lim}$

Where:

- $[Cl^-]_{th}$ is the threshold chloride concentration;
- σ_t' is the ultimate tensile stress;
- w_{lim} is the limit crack width.

The values of $[Cl^-]_{th}$ and σ_t' can vary from a simulation to another, this is why they are represented as an expression in Table 4.3. Unlike w_{lim}, which is fixed by Eurocode to $3\ mm$, and was directly calculated into the ranges.

4.4 Decision analysis

Decisions are based on the assessment of the degradation by measurements in case of inspections, or by estimations in absence of an inspection. Thus, decision may be taken even if there is no inspection performed on the structure.

Let us define τ_D as the inter-decision time interval, and τ_{ins} as an inter-inspection time interval. Let us consider τ_D as constant for the lifetime of the structure, and at every decision epoch a decision must be made. Let us consider that τ_{ins} is a multiple of τ_D, thus $\tau_D \le \tau_{ins}$.

As a result, the general scenario of decision that is proposed is the following, $\forall t > 0$:

- If we inspect at time t, the maintenance decision is based on the CI_n class that is correspondent to the observed values;
- If no inspection is carried at time t, the maintenance decision will be based on an estimation of the probability of belonging to a CI_n class or range.

Where t is a decision epoch, and n is the CI class / $n \in [0-9]$.

Therefore, the challenge here is to estimate this probability of belonging to a CI_n class when inspections are not available. To this aim, we propose here to illustrate the evaluation of the probability of belonging to a class CI_n, for three different cases. The difference between the three cases is on two stakes: 1) in terms of inspection plans, that is if ρ and θ share the same inspection rates or don't, and 2) in terms of the definition of the probability of belonging if it's univariate (ρ) or bi variate (ρ and θ).

To vulgarize, the three cases that will be treated in this section are:

Stake 1: Estimate the probability of belonging in a CI_n for $\rho \in [s_{n+1}, s_n]$:

- where ρ and θ share the same inspection plan, treated in section 4.4.1.
- where ρ and θ share different inspection plans, treated in section 4.4.2.

Stale 2: Estimate the probability of belonging in a CI_n for $\rho \in [s_{n+1}, s_n]$ and $\theta \in [q_{n+1}, q_n]$:

- where ρ and θ share different inspection plans, treated in section 4.4.3.

To estimate the probability of belonging to a class, we need to assess the condition of the structure. Therefore, in the next three sections, we will explain the procedure for the assessment of the condition for the three cases identified previously. And finally in section 4.4.4, we present the decision process that need to be carried out at every decision epoch.

4.4.1 Same inspection plan for ρ and θ

In this first case, ρ_i and θ_i share the same inter-inspection interval, *i.e.*, $\tau_{ins\_\rho_i} = \tau_{ins\_\theta_i} = \tau_{ins}$. Where i is the number of phase, $\tau_{ins\_\rho_i}$ is the inter-inspection time of the ρ_i indicator, and $\tau_{ins\_\theta_i}$ is the inter-inspection time of the θ_i indicator.

In Figure 4.4, an example of a possible decisions and inspections plans is represented where we consider that $\tau_{ins} = 2.\tau_D$. In this example, we illustrate unsynchronised plans.

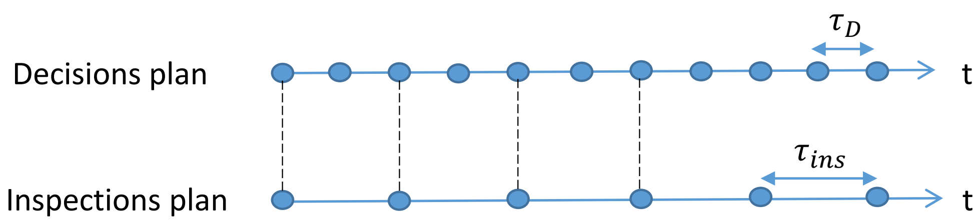

Figure 4.4 Decision and inspection plans for $\tau_D = 2.\tau_{ins}$

The evaluation of the probability of belonging to the class CI_n defined for $\rho \in [s_{n+1}, s_n]$ is not obvious at first. We propose to formulate the corresponding expression of the probability of belonging for a non-maintained structure (this is the simplest scenario compared to all the scenarios that should be considered for the assessment of all the probabilities).

The state of the structure measured at the last inspection at $\tau_{ins} = 2.\tau_D$ is (ρ_1, θ_1), and, we suppose that $\rho_1 \in [s_{u+1}, s_u]$ corresponds to class CI_u. We note $P_{k=2}\big(CI_n/(\rho_1, \theta_1)\big)$ the probability associated to the belonging to class CI_n. This probability can be expressed as a function of the marginal density of the probability of the increment $\Delta\rho$ expressed for two consecutive intervals τ_D.

The marginal density of the probability of the increment $\Delta\rho$ at $\tau_{ins} = 2.\tau_D$ is given by:

$$\begin{aligned}
&\tilde{f}_{\Delta\rho}(x; 2.\tau_D, \rho_1, \theta_1) \\
&\qquad = \int_0^x \int_0^\infty f_{\Delta\rho}(x_1; \tau_D, \rho_1, \theta_1, y_1) \cdot f_{\Delta\theta}(y_1; \tau_D, \rho_1, \theta_1) \\
&\qquad \times \int_0^\infty f_{\Delta\rho}(x - x_1; \tau_D, \rho_1 + x_1, \theta_1 + y_1, y_2) \\
&\qquad \cdot f_{\Delta\theta}(y_2; \tau_D, \rho_1 + x_1, \theta_1 + y_1) dy_2\, dy_1\, dx_1
\end{aligned} \tag{4.13}$$

Under the previous conditions, we will have:

$$\mathrm{P}\big(CI_n \big| (\rho_1, \theta_1)\big) = \int_{s_{n+1}-\rho_1}^{s_n-\rho_1} \tilde{f}_{\Delta\rho}(x; 2.\tau_D, \rho_1, \theta_1) dx \tag{4.14}$$

The expression of the marginal density does not allow a numerical procedure for the assessment of $\mathrm{P}_k\big(CI_n \big| (\rho_1, \theta_1)\big)$ when $k > 2$, especially since it is necessary to integrate all possible maintenance scenarios. Henceforward, we propose to use a Monte Carlo procedure to estimate the probability of belonging to a CI_n class.

In Figure 4.5, the proposed algorithm for this case is summarized.

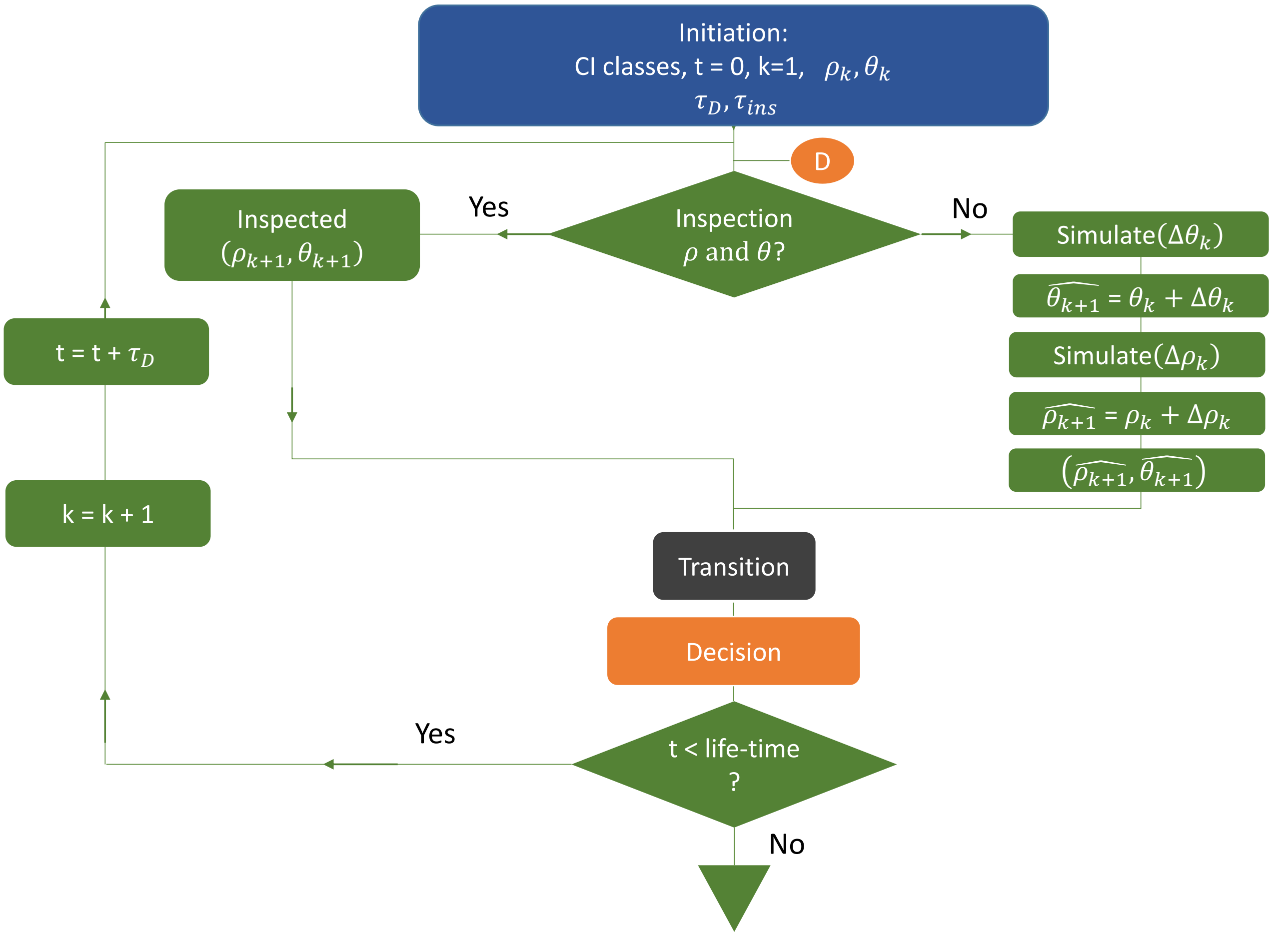

Figure 4.5 Algorithm of the decision model

NB – the simulation of increments is sequential (*c.f.* § 4.2.2). In this algorithm, we have noted that $\Delta\theta_k$ is simulated first then $\Delta\rho_k$. Although this is true for phases 2 and 3, it is not for phase 1 where the sequence is opposite. In this algorithm, and the future ones, we kept the first order, however, in the written program it is taken into account to properly simulate each phase. Here, we are interested in the logic of the algorithm.

The decision process, represented by the box title "Decision" in the algorithm will be treated in section 4.4.4. The aim of this box is to take decisions based on the CI_n class, once identified

The transition process, represented by the box title "Transition" is a test to check if a transition has occurred. In such occurrences, the laws of evolution appropriate to the new phase must be used, although the time passed in the new phase needs to be calculated. Let us define t_s as this time.

The identification of t_s is used to initialise the new phase based on the time that has evolved already, that is $t + \tau_D - t_s$. t_s is calculated by finding the abscissa of the intersection between the degradation level before and after transition, with the threshold line.

In Figure 4.6, the algorithm of the transition box is represented. In Figure 4.7, t_s is shown.

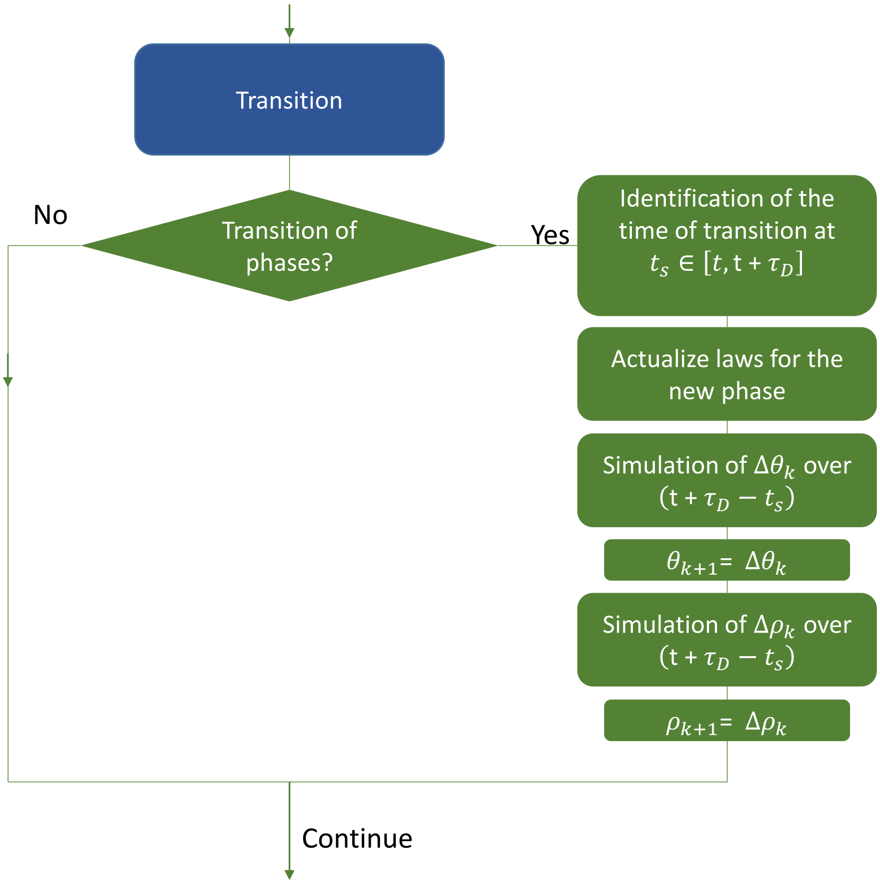

Figure 4.6 Transition algorithm

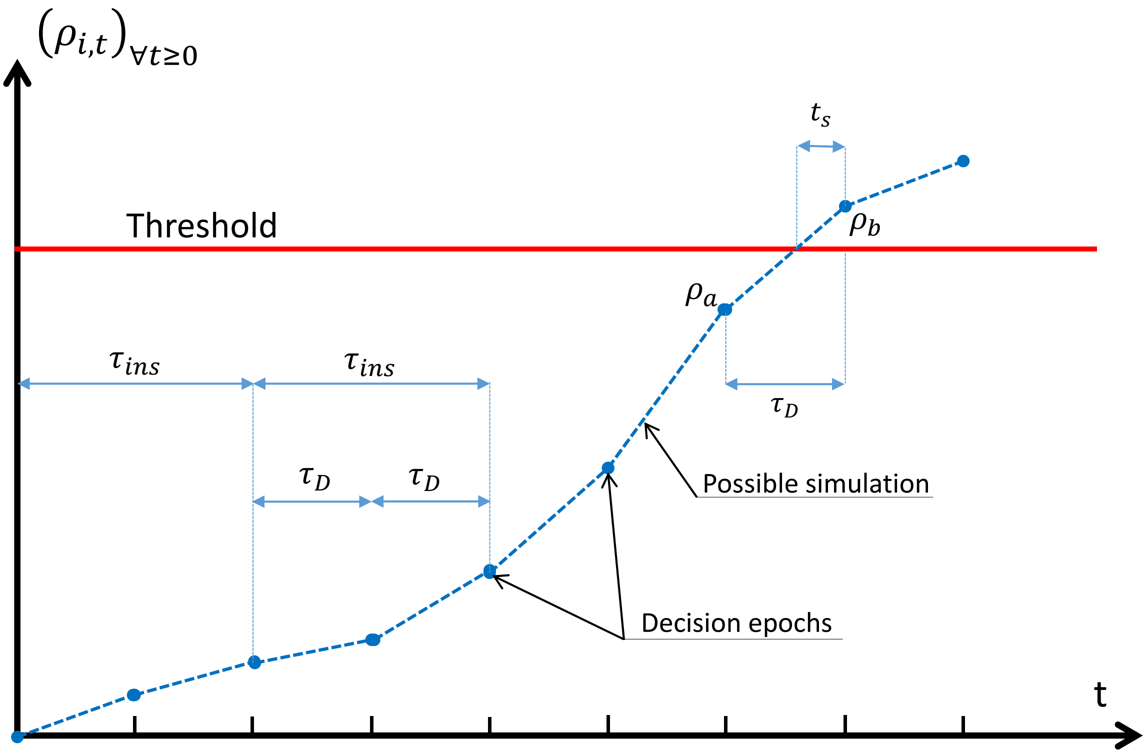

Figure 4.7 Identification of t_s

To illustrate the use of this algorithm, we consider $\tau_{ins} = 3.\tau_D$. Furthermore, we limit the illustration to the third phase of the degradation process, *i.e.*, crack propagation, represented by the crack width and the corrosion current density, respectively ρ_3 and θ_3.

In Figure 4.8, one simulation based on a possible history of inspections is represented. In this figure, we can witness the propagation of uncertainty in reconstructed data when we go farther from a made inspection.

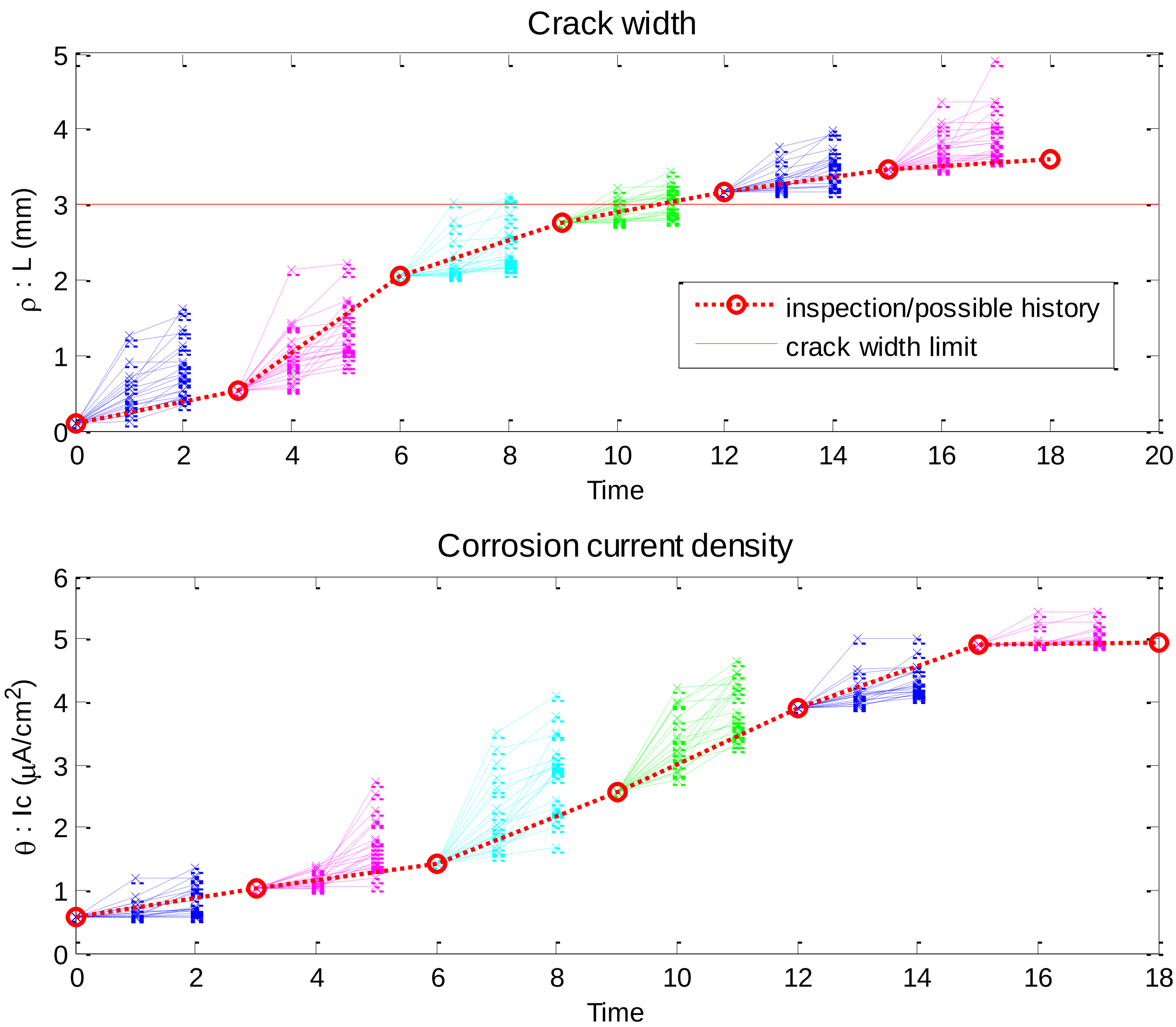

Figure 4.8 Simulation for $\tau_D = 3.\tau_{ins}$

Using these simulations, we can estimate the probability of belonging to a CI_n class.

In Table 4.4, the estimated probabilities are summarized.

Table 4.4 Probabilities of belonging to CI class in the third phase

Time	0	1	2	3	4	5	6	7	8	9	10	11	12
CI	Ins			Ins			Ins			Ins			Ins
0	0	0	0	0	0	0	0	0.01	0.04	0	0.12	0.34	1
1	0	0	0	0	0	0.02	0	0.05	0.18	0	0.7	0.64	0
2	0	0	0.02	0	0.05	0.1	0	0.79	0.78	1	0	0.02	0
3	1	1	0.98	1	0.95	0.81	1	0.16	0.01	0	0	0	0

The highest probability per epoch is distinguished using the red colour. Also, inspected epochs are coloured in grey. Inspections are considered perfect, hence, the correspondent CI_n class is certain ($probability = 1$).

For almost all the epochs, the maximum probabilities of belonging, per epoch, are higher than 0.7. However, for $t = 11$. $\mathrm{P}(CI_1) = 0.64$ and $\mathrm{P}(CI_0) = 0.34$ (*i.e.*, failure). In cases where the detection of failure is of importance to the maintenance policy, a possible decision is to inspect at $t = 11$ in order to be certain of the true CI_n class for this epoch.

Finally, in this first case, we evaluated the probabilities of belonging using the same inspection plans for ρ_i and θ_i. However, it is possible that the two degradation indicators are not inspected at the same time. Therefore, in the next two cases, we consider that ρ_i and θ_i are inspected with different rates, in other terms, $\tau_{ins\_\rho_i} \neq \tau_{ins\_\theta_i}$.

4.4.2 Different inspection plans for ρ and θ

In this case, where we have different inspection plans for ρ_i and θ_i, four cases emerge:

- Both parameters ρ_i and θ_i are inspected.
- Only ρ_i is inspected, as a result, $\widehat{\theta_i}$ is estimated.
- Only θ_i is inspected, as a result, $\widehat{\rho_i}$ is estimated.
- No inspections, as a result, $\widehat{\rho_i}$ and $\widehat{\theta_i}$ are estimated.

Nonetheless, the same idea is used, that is, uninspected value are calculated in order to estimate the probability of belonging to a CI_n class.

In Figure 4.9, the general algorithm is summarized.

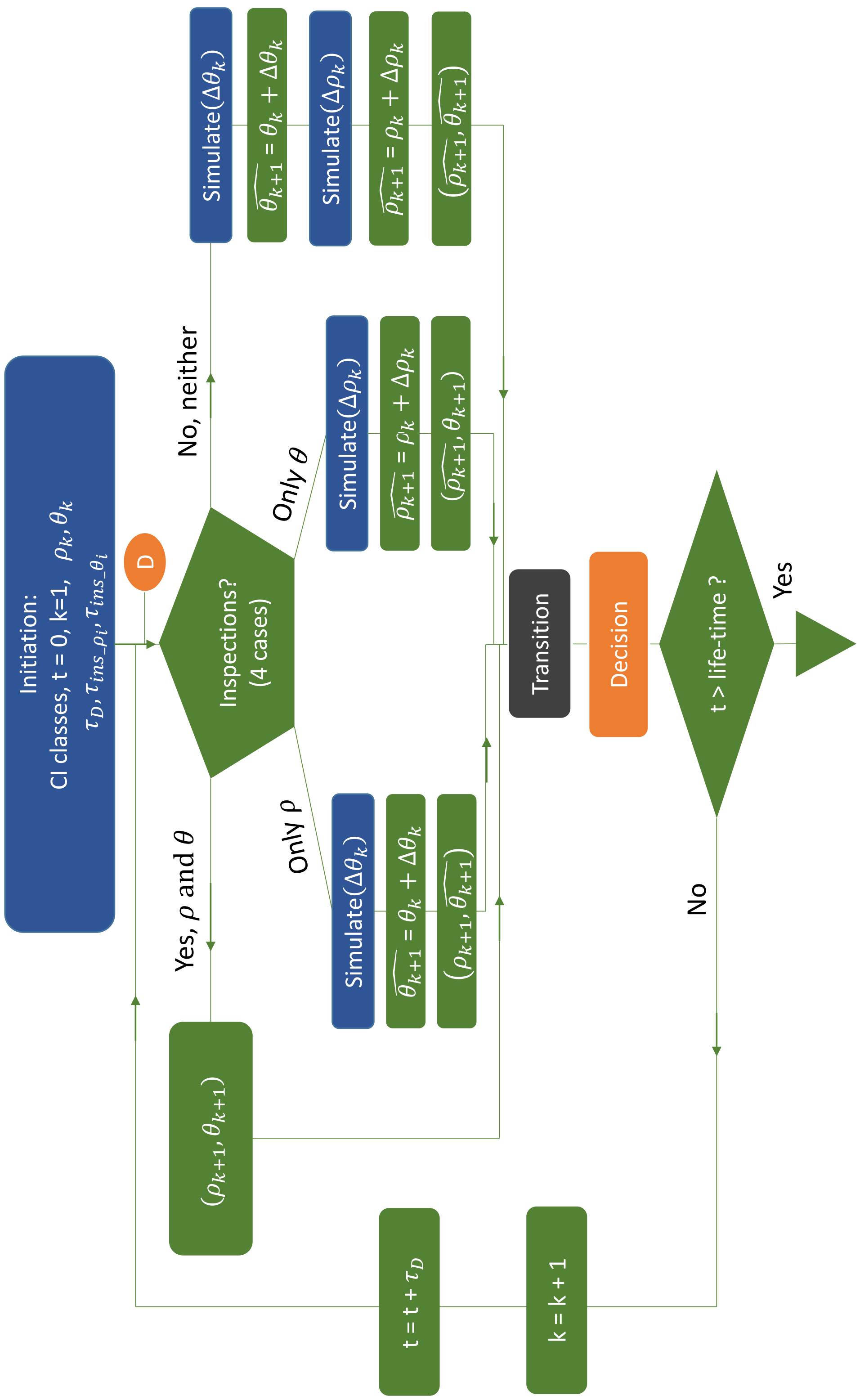

Figure 4.9 General decision algorithm

The pentagon titled “inspections?” at the centre of the algorithm is the test that determine in which of 4 cases mentioned earlier we are.

Similarly to the previous section, we propose to illustrate the use of the algorithm for the third phase of degradation. In this illustration, we consider the following inspection plans:

- $\tau_{ins\_\rho_3} = 3.\tau_D$, inter-inspection interval for inspecting the crack width.
- $\tau_{ins\_\theta_3} = 5.\tau_D$, inter-inspection interval for measuring the corrosion current density.

In Figure 4.10, reconstructed simulations based on a possible history of inspection are represented.

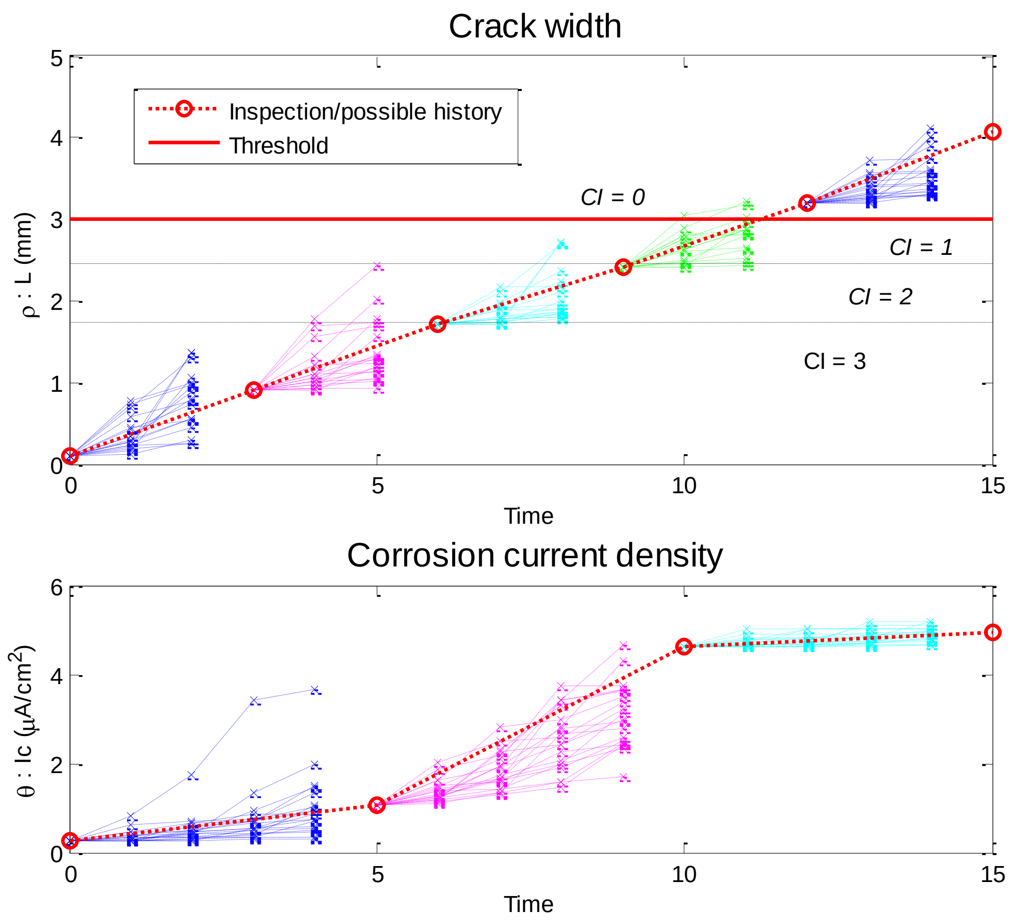

Figure 4.10 Illustration for $\tau_{ins\_\rho} = 3.\tau_D$ and $\tau_{ins\_\theta} = 5.\tau_D$

In this figure, we can notice the three CI classes for the third phase. The simulations of the reconstructed data are scattered on these CI classes, therefore, it is possible to calculate the probability of belonging to a CI class at every decision epoch.

To ameliorate the estimation procedure, we exploit the sequential simulating of the degradation model, *e.g.*, sometimes we need ρ to simulate θ, and vice-versa. Therefore, in order to ameliorate this procedure, the estimation of the probabilities of belonging is based on simulation of increments that reach the inspected values. In example, if we suppose θ is inspected, in order to estimate ρ, we take only the trajectories with a smaller θ at the inspection before.

In Table 4.5, probabilities of belonging to a CI class are summarized. In this table, we can see the highest probability at every epoch coloured in red. Also, we can see the inspection epochs where we can distinguish between the three cases of inspection, these are:

i. Ins (ρ, θ): where both indicators ρ_i and θ_i are inspected (coloured in dark grey);
ii. Ins ρ: where only ρ_i is inspected (coloured in light grey);
iii. Ins θ: where only θ_i is inspected (coloured in medium grey).

In this case, the probability of belonging to a CI class is only based on ρ_i. Nonetheless, we have considered different inspection rates for ρ_i and θ_i. In section 4.4.3, we will keep the different inspection rates; furthermore, we will consider a bivariate approach to evaluating the probability of belonging to a CI class.

Table 4.5 Probabilities of belonging in a CI class

CI	0	1	2	3	4	5	6	7	8	9	10
	Ins (ρ, θ)			Ins ρ		Ins θ	Ins ρ			Ins ρ	Ins θ
0	0	0	0	0.01	0.05	0	0.11	0.61	0.93	0.999	1
1	0	0	0.01	0.02	0.07	0.04	0.38	0.34	0.07	0.001	0
2	0	0.01	0.05	0.1	0.25	0.8	0.51	0.05	0	0	0
3	1	0.99	0.95	0.88	0.63	0.16	0	0	0	0	0

4.4.3 Bivariate decision process

A more complex approach is to evaluate the probability based on information issued from both indicators, ρ and θ. In the previous cases, the evaluation of the probability of belonging to a CI_n was univariate and calculated for $\rho \in [s_{n+1}, s_n]$. Whereas in this case, the evaluation of probability is bivariate, calculated for $\rho \in [s_{n+1}, s_n]$ **and** $\theta \in [q_{n+1}, q_n]$.

Furthermore, in the previous cases we defined the CI classes as intervals on ρ_i, whereas here, the classes are defined as zones on a bi-dimensional graph, where the horizontal axe is ρ_i, and the vertical axe θ_i.

To construct the zones, we discretise the θ_i in the same way as for ρ_i (§ 4.3.2.4, Figure 4.10). Then we connect the discretised axes, at the respective discretized points, two by two, to form the condition zones.

Then, using the same algorithm used for different inspection rates for ρ_i and θ_i (Figure 4.9), we estimate the increments.

The second step is to scatter all simulated or inspected couples (ρ_i, θ_i) onto the discretised plan, let's call it the decision graph. For every epoch, we have a different decision graph.

In order to illustrate the use of a decision graph, we propose to take the same example of section 4.4.2. Since a decision graph is drawn for each epoch, for the sake of this example, we consider the epoch #4. Then, we scatter all the simulated couples (ρ_3, θ_3) at epoch #4 on a plan that is discretized as explained previously. In Figure 4.11, the decision graph, the 4 CIs classes, and the scattered couples are represented.

On this figure, we can see the probability of belonging to a CI zone. We note that for epoch #4, neither ρ nor θ are inspected, therefore, the dispersion of couple on the graph are in both directions. The lower and left bond on the scattered couples are a direct consequence of the monotone nature of the SDGP.

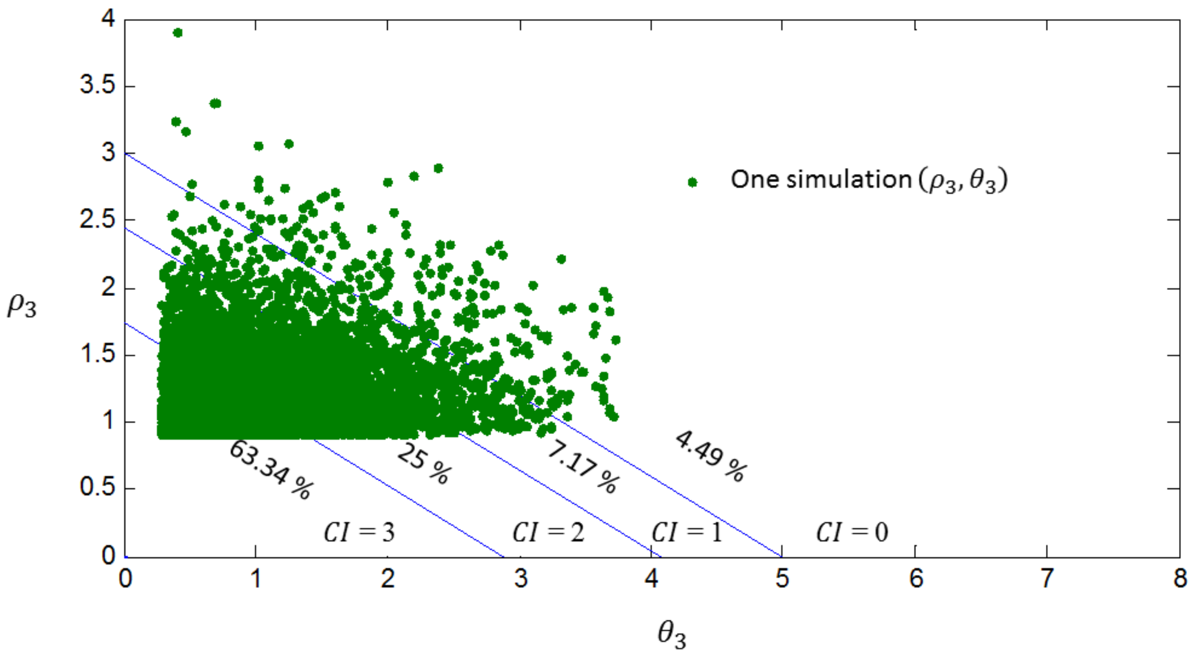

Figure 4.11 Decision graph for the 4th epoch

The choice of CI_nclasses remains entirely arbitrary in this study, whether it was bivariate or univariate. The proposition of more complex models can be thought of. Here we propose three types of potential discretization for the CI_n:

i. State-based CI_n classes (*c.f.*, § 3.5.2).
ii. Non linear CI_n zones.
iii. Increase the number of CI_n classes for more possible actions.

4.4.4 The decision box

The decision process, represented by the box title "decision" in the previous algorithms, can be approached in two approaches: a univariate or a bivariate way. The difference between the two approaches is in the evaluation of the probability of belonging to a class

In sections 4.4.1 and 4.4.2, a univariate approach was used where the probability of belonging is evaluated for $\rho \in [s_{n+1}, s_n]$. Whereas, in section 4.4.3, the probability is evaluated for $\rho \in [s_{n+1}, s_n]$ and $\theta \in [q_{n+1}, q_n]$.

Once this probability is evaluated, the decision process is the same.

As mentioned briefly earlier, a minimum probability of belonging is considered to define an inspection decision to ameliorate the CI_n estimation, or a maintenance decision (a maintenance action is not necessarily a reparation one). A decision is attributed to every CI_n.

For the purpose of this example, we define a minimum probability equal to 0.66.

Furthermore, if a reparation action is carried out, its effect needs to be taken into account in the degradation model; that is, to update the laws of evolution and the state accordingly with the reparation action (*c.f., 4.3.2*).

In Figure 4.12, the decision algorithm is represented.

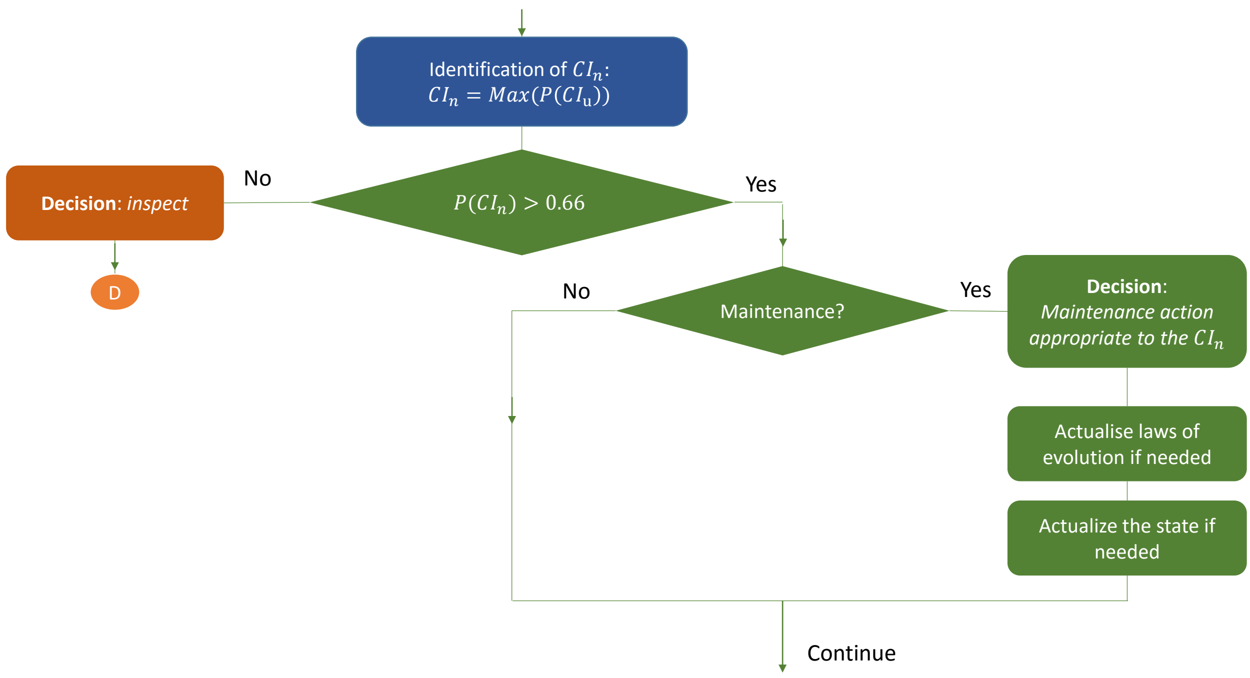

Figure 4.12 Decision process

4.5 Illustration on maintenance policies

In this section, we will illustrate the use of the meta-model in a maintenance context by defining maintenance policies. A maintenance policy in this context will provide an assessment of life-cycle costs and condition indexes CIs. These assessments will help compare policies, and in the future help to integrate the meta-model into an optimization framework. The optimization part will not be addressed in this thesis, and is kept to future works of the ***SI3M*** project (that this thesis is part of).

Two maintenance policies are considered in this study: a preventive maintenance policy and a corrective maintenance policy. The first aims to prevent corrosion from starting, and the latter repairs after failure. We will assume three lifetimes in order to compare the two policies in terms of CIs and life cycle cost. The three lifetimes are 50, 75 and 100 years.

In the aim to assess lifecycle costs and CIs, to take maintenance actions, and to model the effect of maintenance actions on the model, we will exploit the tri-phased bivariate meta-model that represents the three phases of chloride-induced corrosion in submerged reinforced concrete.

Each maintenance policy aims to repair the structure for a chosen CI. Maintenance costs and CIs are evaluated by carrying out Monte Carlo stochastic simulations under the Matlab® environment. The outputs of the simulations are inspections data, and the history of the structures that will be represented by the loss of steel. The decision of carrying out a repair action is based on these outputs, and depends on the maintenance policy and the extent of damage.

For this example, we consider only the concrete replacement repair methods; CR1, CR2 and CR3. This choice was made since we dispose of real costs for the repair of a marine harbour in Saint-Nazaire, France (Srifi 2012).

In Table 4.6, the costs of maintenance actions and inspections per phase are summarized.

Table 4.6 Costs of maintenance actions and inspections per phase for CR

Phase	I – Diffusion of chlorides	II – Initiation of corrosion	III – Propagation of the crack
Maintenance action	CR1	CR2	CR3
Maintenance cost ($€/m^2$)	263.2	323	353.4 +2000 halt cost
Inspection ($€/m^2$)	25	25	10

When it is required to carry out a CR3, where we replace concrete and steel, usually, additional substantial costs must be considered. These costs are due to the halt of the operation of the harbour, let's call them *halt costs*. Therefore, for CR3, 2000 $€/m^2$ are added to the original cost of the maintenance action, bringing the total cost of CR3 to 2353.4 $€/m^2$.

In order to illustrate the benefit of the approach in a maintenance context, we propose two maintenance policies. First, a preventive maintenance policy where maintenance is carried out before the initiation of corrosion, aimed at a $CI = 7$. The aim of this policy is to prevent the initiation of corrosion, and therefore carry out a CR1 in the first phase. However, sometimes inspections misses $CI = 7$, and corrosion starts. This case will be dealt with and discussed in section 4.5.1. Inspections are considered perfect, and the inter-inspection time is fixed to 5 years.

Secondly, we propose a corrective maintenance policy where maintenance is carried out after failure, *i.e.*, $CI = 0$. The maintenance action in this case is a CR3, generating substantial over costs.

For each policy, 10000 simulations are carried out in order to determine the expected costs and the CIs. Total costs for each policy are estimated at the present time, without considering a discounting factor. Since we focus on existing structures, construction and salvage costs are not included in this analysis.

For this study we consider the following parameters:

- ***1^st^*** phase***:***

  $a_1 = 2.8$, $a_2 = 4.2$, $a_3 = 0.15$, $a_4 = 0.15$, $a_5 = 0.2$, $a_6 = 0.1$, $a_7 = 0.15$, $\beta_{\rho_1} = 0.3$, and $\beta_{\theta_1} = 0.3$.

- ***2^nd^*** phase***:***

  $b_1 = 3.1$, $b_2 = 3.2$, $b_3 = 1$, $b_4 = 0.15$, $b_5 = 0.25$, $a_6 = 0.05$, $a_7 = 1$, $\beta_{\rho_1} = 0.2$, and $\beta_{\theta_1} = 0.3$.

- ***3^rd^*** phase***:***

  $c_1 = 2.5$, $c_2 = 4$, $c_3 = 1$, $c_4 = 1.2$, $c_5 = 0.4$, $c_6 = 0.9$, $c_7 = 1$, $\beta_{\rho_3} = 0.2$, and $\beta_{\theta_3} = 0.3$.

In Figure 4.13, the maintenance polices and inspection plans are summarized.

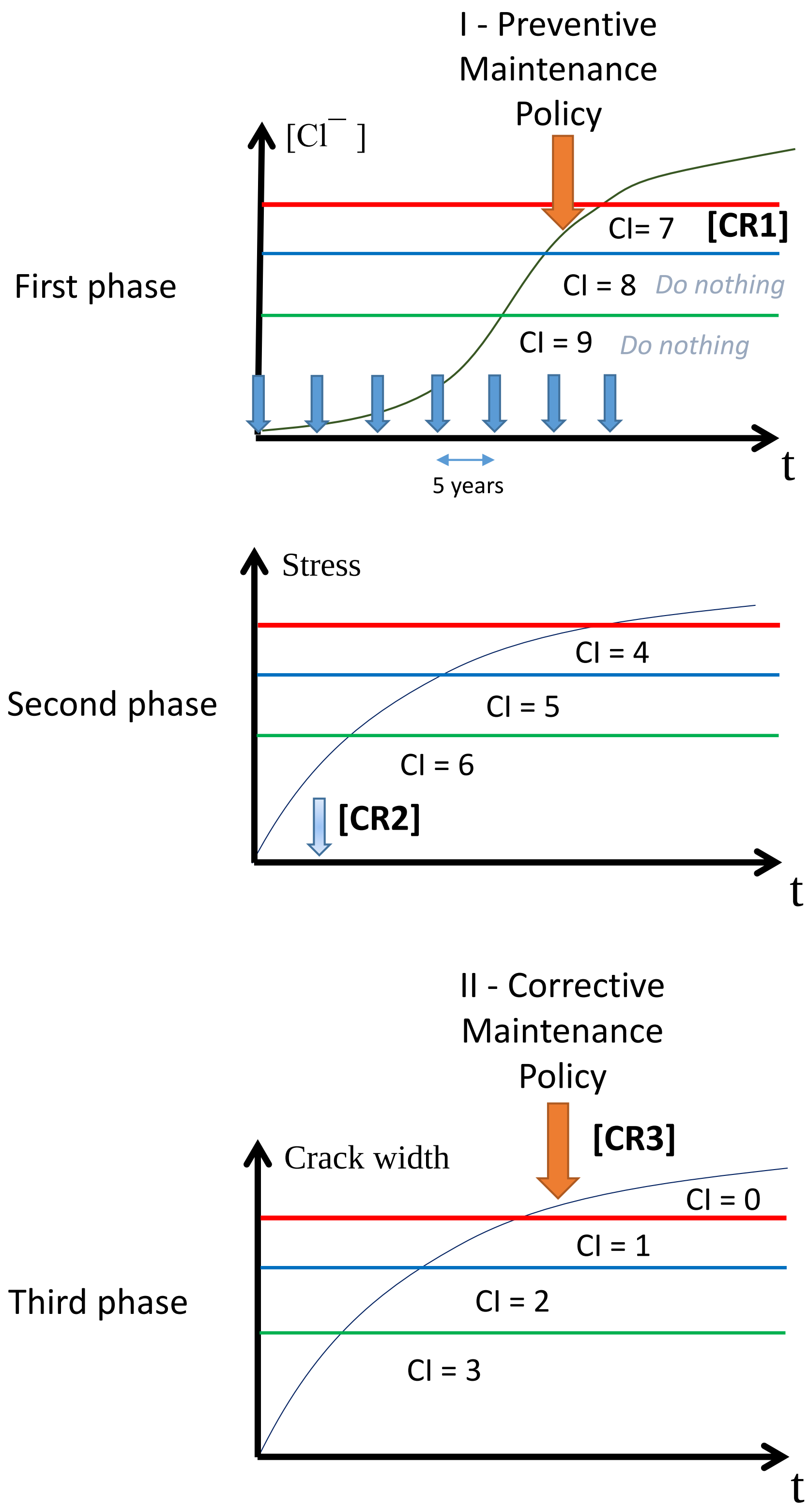

Figure 4.13 Maintenance planning

4.5.1 Preventive maintenance policy

The preventive maintenance policy (PM) aims at repairing the structure for a target $CI = 7$ (*i.e.*, just before the end of the 1st phase). The intention from this policy is to prevent the initiation of corrosion, therefore, conserve the steel intact of corrosion.

However, this intention is not always satisfied. In fact, it is possible to miss an inspection that is in the $CI = 7$ range. The two principal actors that can contribute to the missing of the aimed CI are: a) the inter-inspection time, and b) the relatively small size of the $CI = 7$.

In this case, we have to inspect appropriately for the second phase, ρ_2 and θ_2. The inspection serves to make sure that indeed the corrosion has started, and the structure is in the 2nd phase. Also, the inspections help to track the evolution of the corrosion in terms of loss of steel, helping to establish a continuity for the degradation process in case the missing of the aimed CI occurred more than once. Later in this paragraph, we discuss the modelling of steel loss in the concrete.

The second consequence of missing $CI = 7$ is that a CR2 is required instead of the originally planned CR1 generating more costs. These costs are added into total maintenance costs, and will be called *over-costs*. Now, if the corrosion was active numerous times during the lifetime of the structure, the loss of steel might be significant enough to consider a CR3 (concrete and steel replacement) generating substantial halt-costs to the over-costs. The triggering of a CR3 is dependent on the amount of lost steel; when the corrosion has consumed so much of the steel to a degree that it cannot play its intended structural roles and needs to be replaced.

Table 4.7, the costs and the CIs for three different life times of a structure maintained through a preventive policy are summarized. The Condition Index in the tables is the mean of all the inspections' condition indexes.

Table 4.7 Costs and Condition index for a preventive maintenance policy

Lifetime (years)	**50**	**75**	**100**
Inspections $(€/m^2)$	320	466	622
Maintenance $(€/m^2)$	870	1330	1815
Total cost $(€/m^2)$	1182	1765	2424
Annual cost $(€/m^2/year)$	23.9	24	24.3
Condition Index	8.21	8.18	8.14

This table will be used later in section 4.5.2 in order to compare the two maintenance policies.

4.5.1.1 The event of missing $CI = 7$ and its consequences

The probability of missing $CI = 7$ is defined as the probability of having two consecutive inspections in a way that, the first inspection find a $CI \geq 8$, and the immediately succeeding inspection find a $CI \leq 6$.

In Table 4.8, the probability and cost of missing $CI = 7$, and the probability of triggering CR3 are summarized.

Here we can experience the need of optimization, in example.

Table 4.8 Probabilities and over-costs of missing $CI = 7$

Lifetime (years)	**50**	**75**	**100**
Over-cost (€/m²)	147	223	320
Percentage from total cost	12%	13%	13%
Prob. of missing $CI = 7$	0.263	0.265	0.283
Prob. of triggering CR3	0	0	0

Over-costs represent about 13% of the total cost for the three lifetimes. This is a high cost that can, and needs to be, cut down. In order to do so, the decision maker has a choice between two options:

i. Modify the ranges of the condition indexes by increase the height of $CI = 7$
ii. Intensify the inspection rate.

If the zone 7 was bigger, it is more probable to inspect in the aimed CI range rather than missing it and as consequence activate corrosion. But, if the range is largely increased, it is also probable to trigger an early, unnecessary, maintenance action.

Now, if the inspection rate is increased, this will also generate additional cost, however, they may be less costly than increasing the range of $CI = 7$. A more suitable inspection planning technique would be a risk or condition-based inspection approach, where the time of the next inspection will be based on the current level of degradation. In example, if we are in a $CI = 8$, we lower the inspection rate to an inspection per year. In this way, we lower considerably the probability of missing the aimed CI, and with it, we lower the over-costs.

4.5.1.2 Loss of steel

In the previous example (Table 4.8), CR3 was never triggered. Nevertheless, in this section we will explain how the loss steel is modelled. This explanation can serve as the basis for combining covariates or other indicators with the SDGP.

In the PM context, we aimed to carry maintenance at the end of the first phase ($CI = 7$). As seen earlier, it is possible to miss the end of the phase. In consequence, corrosion will be activated and the corrosion reaction starts. The main elements of this reaction are chloride, hydrogen, oxygen and iron. A product of the corrosion reaction is rust, material that is less dense than the steel that occupied the place in the concrete. Therefore, once the rust fill the pores surrounding the steel, it will start to exert internally on the concrete. This internal force is expressed as internal stress and is the main reason that the concrete starts its cracking.

The lost steel from the corrosion reaction is not recovered neither by CR1, nor CR2. Therefore, if this happens many times, the lost in steel may become significant. For a cumulative loss of 30% or more, we consider that the steel will not be able to perform structurally (*e.g.*, adhesion with the concrete, reinforce the concrete in face of bending and traction). In these cases, a concrete replacement of the third order is required (CR3) in which corroded steel are replaced along with the contaminated concrete.

Therefore, every time the corrosion is activated, we estimate the duration of active corrosion, as well as the corrosion current density. Using the estimates we can estimate the loss of steel.

Using Faraday's laws of electrolysis, we can express the loss of material as function of the corrosion current density:

$$\frac{\Delta s}{\Delta t} = 3268 \frac{i_{corr} M}{zFq} \; mm/year \qquad 4.15$$

Where,

- i_{corr} is the corrosion current density given in A/cm^2.
- z is the number of electrons dreed by the corrosion reaction.
- M is the atomic mass (g/mol atoms).
- F is the Faraday's constant (96 485).
- q is the density of the metal ($g/cm3$).

In Figure 4.14, the process of steel loss is illustrated showing a triggering of the CR3 (which was not the case in our example, matter of fact, the lost in corrosion was insignificant). Therefore, this figure is for explanation purposes only and not numerically related to the previous example.

In the figure, we can see the activity of the corrosion in accordance to each phase. Two phases are present in the PM context, the first where $[Cl^-]$ the parameter of interest, and the second phase is where stress (internal) is. The corrosion current density is null for the 1st phase (corrosion inactive), and non-null in the 2nd phase (corrosion active).

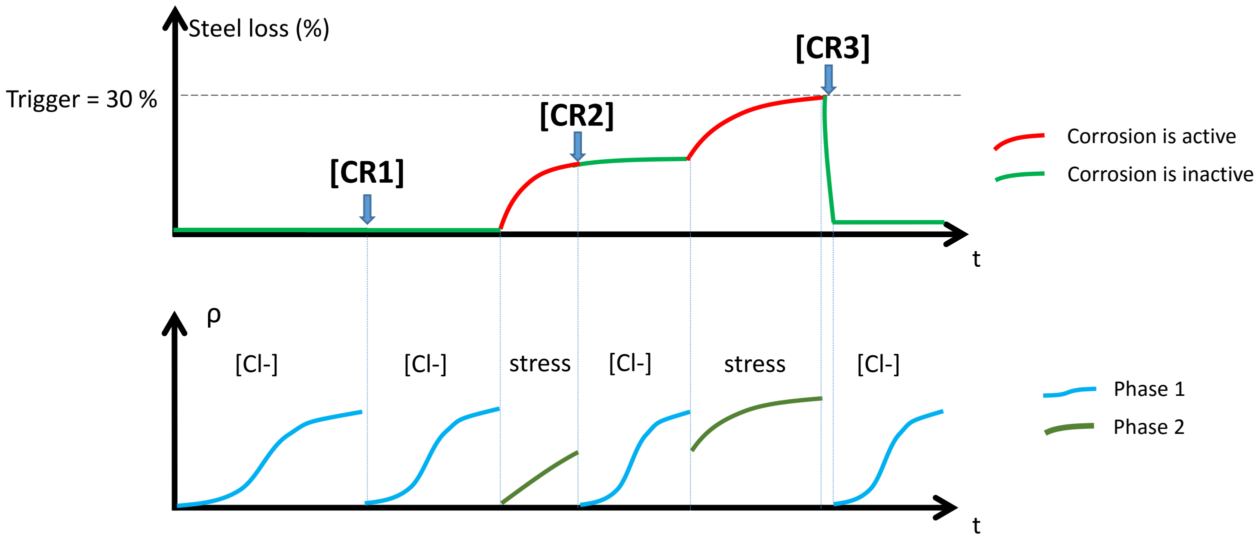

Figure 4.14 Loss of steel process used to trigger CR3

4.5.2 Corrective maintenance policy and comparison with PM

The corrective maintenance policy (CM) aims at repairing the structure after failure, *e.g.*, severe cracking (crack $> 3mm$). Therefore, the CM policy performs a CR3 as soon as a $CI = 0$ is detected.

In this policy, the inspection plan is different from the previous one. Here, we are only interested in the crack width; when the concrete starts cracking, and when it reaches failure. Hence, the inspection plan here is based on 5 years inter-inspections for the crack width only. The first effect of this decision is the inspection cost, where inspecting for the third phase is cheaper (visual) than the two first phases.

Table 4.9 provides the costs and the CIs for three lifetimes.

Table 4.9 Costs and Condition indexes for the corrective maintenance policy

Lifetime (years)	*50*	*75*	*100*
Inspections (€/m²)	136	200	263
Maintenance (€/m²)	2350	4142	5224
Total cost (€/m²)	2484	4341	5490
Annual cost (€/m²)	50	58	55
Condition Index	6.3	5.86	5.89

Now, we compare the results in Table 4.7 and Table 4.9. We can see that the CM compromises on the CI when compared with the PM. The PM preserve a higher $CI \approx 8$, whether the CM has a $CI \approx 6$. In terms of performance index, the PM is better that the CM.

From a cost point of view, the CM is cheaper in terms of inspection (60% less) but substantially more costly on maintenance action ($250 + \%$ more) due to halt costs ($+2000$ €$/m^2$). Consequently, the PM is cheaper by **half** than a CM.

As a result, the PM is by far better that the CM for this single application. However, we emphasize on the objective of this study, that is to show the applicability of the meta-model in a maintenance context, which was done here.

4.6 Conclusion on maintenance analysis

In this chapter, we illustrated the potential use of meta-models in a maintenance context, in particular for multi-phasic degradations. The illustration was made through applications and analysis.

First, we started with the construction of the meta-model for three phase, chloride-induced corrosion. A degradation model must cover the lifetime of a structure if is to be used in a maintenance context.

Second, an approach to model the effect of a maintenance action effect on the degradation model was presented. In this approach, a maintenance action's effect is modelled by introducing parameters with physical meanings into the shape functions of the degradation model, called "maintenance parameters". This gives us the ability to compare maintenance actions in terms of effect on the degradation model, hence, on the degradation process's prediction.

Third, a proposed approach to make decisions based on the probability of being in CI class was suggested. That is, condition indexes used to quantify the performance of the structure. For the evaluation of the probability three cases were considered.

Forth, using the meta-model, we estimated a life-cycle cost and a performance index for two proposed maintenance policies, paving the road to consider a multiple objective optimization process in future works, *e.g.*, maximization of structural performance while minimizing the life-cycle cost.

This chapter remains introductory and an illustrative chapter on the use of meta-models in a maintenance context. However, the mathematical formulation of the meta-model gave a lot of "controlled freedom" to the model, that allowed for applications to be carried out easily and with no extraordinary difficulties, making the meta-model's use in a maintenance context advisable.

This chapter opened many doors to future work and studies. These ideas, offer a potential to further study the applicability of meta-models in a maintenance context.

Chapter five

Conclusions and future works

– Abstract –

In this thesis, we developed a probabilistic degradation modelling technique for maintenance purposes of deteriorating structures and infrastructures. In particular, the technique covers the modelling of degradation that can be expressed by accumulative measurable degradation indicators, accessible through inspections, especially NDT. The proposed technique, called meta-models, is based on a formulation of state-dependant stochastic processes to model the physical indicators. The three fundamental chapters of this thesis are parts of the SI3M project that aims to develop maintenance management tools based on meta-models for deteriorated structures and infrastructures, especially marine ones. In this final chapter, we summarize the conclusions of this thesis and we propose future works.

5.1 Conclusions

In the opening of this thesis, we emphasised the importance of maintenance in today's society, and the necessity to tackle its challenges to ensure a higher performance and safety of structures and infrastructures, with a lower cost and/or other objectives. In the introductory chapter, we discussed some of the concerns of maintenance and outlined four questions that were going to be addressed in this thesis (page 13). Hence, this chapter is organized in a way to answer to these questions based on the findings of this thesis.

The first question was **"How to improve the evaluation, modelling and prediction of degradation for maintenance purposes?"**

This question was analysed in two parts: part one, what are the new purposes of maintenance, and part two, what qualities are required from the degradation model to be able to respond to these purposes.

First of all, maintenance's elements have extensively evolved in the last two decades. In Chapter 2, while reviewing inspections (§ 2.3), maintenance management systems (§ 2.2) and decisions in maintenance (§ 2.3.4), we sensed the need to extend the characteristics of degradation models in a maintenance context (§ 2.4).

The requirements of degradation models have evolved from their classical role in capturing and simulating the physics of the degradation, to where they now need to be able to reliably predict the degradation using all available information, to take into account uncertainties, to include data issued from NDT inspections, and finally to allow their integration in complex condition based maintenance platforms (§ 2.4.1).

Conventional approaches to model degradation can be classed in two categories (Frangopol et al. 2004). On the one hand, we found mechanist approaches that simulate the underlying physical laws, also known as white-box or physics-based models (§ 2.4.3). These models are conceived for structural design in civil engineering, thus, suffer from a lack of applicability in a dynamic maintenance context, especially when it comes to integration of NDT and uncertainty. On the other hand, there are pure probabilistic models based on statistical quantities (§ 2.4.2). These models need no understanding of the physics of degradation and generally focus only on the time of failure with complete disregard of what happens before. Therefore, they give no useful information for civil engineers on the evolution of degradation, and their use in a CBM policy is not adapted.

In the proposed approach, we aimed to keep what is "good" in probabilistic models, *i.e.* accessibility to NDT and uncertainty, and what is "good" in physics-based model, *i.e.* a physical meaning. This combination of "good" qualities has resulted in what is known as data-driven models.

Degradation processes in civil engineering are generally multiphasic, multi-variate and non-stationary. In this thesis, meta-modelling responded with efficiency to the concerns surrounding degradation modelling in a maintenance context. The multiphasic challenge (§ 4.2) was resolved by proposing a uniformed approach to model the degradation (§ 3.3.1), where for each phase of a deterioration process we look into and choose the appropriate degradation indicators (§ 3.2.1), and then model them using state-dependant stochastic processes (*e.g.* §3.3.2). Secondly, in this thesis we were interested in multivariate degradation models where non-stationarity was modelled using continuous bivariate state-dependant gamma processes (§ 2.5.3) for the mathematical advantages of the gamma trend and the state-dependant approach (§ 2.4.4). The construction of the state-dependent degradation model was discussed within the case of a submerged concrete structure subject to chloride-induced corrosion (§ 3.2).

To conclude, the meta-modeling approach aims to model a degradation pathology with a global vision to address the challenges in terms of structure maintenance. It then allows us to guide the selection of physical indicators to respond to both the characterization of each phase, and also to ensure a continuity in terms of degradation assessment across the lifecycle of the structure.

The second question was '**How to be realistic? What to do with missing inspections and lost information?**'

Incompleteness and irregularities of databases and inspections are inseparable from any realistic situation in civil engineering (§ 2.3.3). The model was tested in situations where missing, censored or truncated values were considered in the database (§ 3.3.3.2 and § 3.4.3), and the solution was by means of developing an extension to the classical Maximum Likelihood Estimation or MLE process (§ 3.3.3.1), that is the Stochastic Estimation-Maximization or SEM algorithm (§ 3.3.3.2). Furthermore, an interesting observation is made in the case where heterogeneous databases were included in the estimation process (§ 2.3.3.2), and the process was remarkably improved (§ 3.4.2). These results portray the model's ability to respond to realistic situations. However, further examination must be carried out in the case of heterogeneity.

The third question was '**How to update and model the effect of a maintenance action after a decision?**'

Major advantages of using the gamma trends are that their parameters identify physical tendencies and they exhibited facility in terms of modelling mutual dependencies of processes, directly in the shape function. In fact, the state-dependant shape functions of the gamma process controlled the size of increments, hence, the speed of degradation. As a consequence, to model the effect of a maintenance action on the degradation model, maintenance parameters were integrated into the shape functions (§ 4.3.2.1). This approach has shown the ability to model the effect of a maintenance action on the speed of the evolution of the pathology (§ 4.3.2.3). Also, a maintenance action can modify the level of degradation. The state-dependent meta-model benefits from the Markovian property, as a consequence, the effect on the level of degradation is modelled by giving an appropriate after-maintenance value for the degradation level (§ 4.3.2.2).

The fourth question was '**How to make the best decision throughout the operation time of the system?**'

To answer this question we thought about the challenges that it raises. First, in the operation time of the system, inspections might target different physical indicators, thus, information on the system might not be of the same nature at every step of the way. Second, to take decisions based on the condition of the system, we need to define ranges that limit these condition levels so we do not have an infinite number of possible decisions. Hence, for every range we can allocate a decision. Regarding the first concern, when an inspection is missed, an estimation of the condition must be carried out. This estimation is calculated using the meta-model to simulate increments reaching the missed inspections (§ 4.4). Regarding the second concern, condition indexes were proposed to allow for the degradation to be in 10 discrete classes (§ 4.3.3).

Finally, the performance of the model was evaluated via applications to Eurocode 2 (§ 3.5.1), risk management (§ 3.5.2) and maintenance policies (§ 4.5). The results portrayed the model's robustness and mathematical tractability that motivates further the research on this type of approaches towards risk-based decision-making and ultimately maintenance optimization.

To conclude on the use of meta-models for the probabilistic modelling of maintenance and degradation, we found an advantage in using meta-models when addressing two main stakes:

- First, the ageing model description with a physical meaning of the main probabilistic trends and couplings of the inputs (NDT assessment) and outputs (decision parameter). With this approach, we tackle a key issue: the scourge between more and more complex physical models and the increasing complexity of NDT results modeling and assessment (decoupling, fusion…) with heterogeneous developments between these two scientific fields;
- Second, in a CBM context, the simple description, flexibility, calibration and statistical calculation make this model easy to implement and beneficial to utilize in a risk management framework. The evaluation of these meta-models is done through state-dependent stochastic processes using information given by NDT. The idea is to facilitate the transfer between the available information and the model.

5.2 Future works

It has been said multiple times in this thesis that the ultimate objective of this research is maintenance optimization. To this aim, further maintenance and degradation modeling concerns must be addressed. Many ideas caught our attention and deserve further thinking:

i. In chapter 4, we estimated a life-cycle cost and a performance index for proposed maintenance policies, paving the road to consider a multiple objective optimization process in future works, *e.g.*, maximization of structural performance while minimizing the life-cycle cost. Therefore, the next step would be to set up and define a multi-objective optimization problem.

ii. The study presented here is not based on field data. Although we are confident in the philosophy of the approach, a study to validate the compatibility of the proposed approach with degradation phenomena studied in laboratories, especially regarding offshore structures, must be carried out to ultimately put the approach into practice using real databases.

iii. In the current meta-model, the state-dependency was exclusively governed by the shape functions (α), while the scale functions (β) were considered as constant (§ 3.3.1). Hence, knowing that the integration of β offers an additional degree of liberty, *i.e.*, we can more easily govern the mean and variance. It would be interesting to go further on the challenges that might arise when modeling β as state-dependent as well. Will the construction approach change? What about the understanding of the meta-model, will the increments' speed be controlled only by α, or by a combination of α and β? What happens to the maintenance parameters?

iv. The correlation between physical indicators needs to be studied as well. For example, in the third phase, in the literature there are no clear correlation studies between the corrosion current density and the crack width, but if there was a correlation, how do we take it into consideration in the model? How do we identify the corresponding parameters?

v. A hypothesis that was taken in this thesis was fixed time steps ($\tau = cte$) for the simulation of the model. The model is not infinitely-divisible (§ 2.5.2), hence, what will happen to the simulation of the model if τ changes with time and/or state? How important is the infinite-divisibility property for the optimization procedure?

vi. So far, we have focused the construction of the policy without considering the spatial variability in degradation (problem of geographical positioning of the defect). An inspection is usually performed on a limited area or following a defined mesh. Therefore, it cannot account for the general state of the structure. A perspective of this work would be to take account of the spatial variability, to study the performance of NDT, and to examine and investigate different temporal and spatial inspection plans.

vii. Finally, in order to valorize and allow for this meta-modelling approach to address real life maintenance problems, it is important to develop easy-to-use or commercial software.

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PÔLE DE RECHERCHE ET D'ENSEIGNEMENT SUPÉRIEUR

UNIVERSITÉ DE NANTES

Thèse de Doctorat

Boutros EL HAJJ

Modélisation Probabiliste de Dégradations Multiphasiques pour l'Optimisation de la Maintenance d'Infrastructures en Génie Civil

Probabilistic Modelling of Multiphasic Degradations for Maintenance Optimization of Infrastructures in Civil Engineering

Résumé

Notre société est face à des enjeux importants en termes de maintenance: digues, structures portuaires, ouvrages d'art, navires, avions. Les futurs champs d'éoliennes offshore, dans un environnement très difficile pour les matériaux, seront eux aussi soumis à des mécanismes d'altération important. Au vu des conséquences d'une défaillance, une surveillance doit être opérée sur toute la durée de vie. La maintenance est un élément fondamental pour garantir un niveau de sécurité visé. Elle est conditionnelle car elle dépend des résultats de l'instrumentation in-situ ou des contrôles ponctuels. Elle doit notamment conduire à inspecter au bon moment et avec la meilleure technique dans un contexte de budget limité : l'optimisation est nécessaire. Les techniques de contrôles non destructifs (CND) offrent des potentiels de tout premier plan mais sous exploités. Par ailleurs les incertitudes concernant le matériau, l'environnement et la mesure CND ne sont pas prises en compte car il existe un fossé entre les modèles mathématiques efficaces d'optimisation en contexte incertain et les modèles de dégradations probabilistes. La raison principale est que ces derniers ont été élaborés pour la conception des structures et non pour intégrer les mesures CND ou être couplés avec des méthodes d'optimisation. La thèse vise à proposer une nouvelle stratégie de maintenance pour les structures dégradées à partir de résultats CND via des méta-modèles. On entend par méta-modèles des modèles à faible nombre de paramètres reposant sur l'expertise physique et la pertinence probabiliste d'une part et sur les indicateurs de dégradation et de durabilité directement accessibles à partir de contrôles non destructifs CND d'autre part. La thèse propose une modélisation de phénomènes multiphasiques de dégradation du béton armé reposant sur des processus stochastiques non-stationnaires dépendant de l'état.

Mots clés Modèle de dégradation probabiliste, Maintenance, processus gamma, management des risques, prise de décision, CND, maintenance conditionnelle, dégradation non stationnaire, processus stochastique dépendant de l'état.

Abstract

Our society is facing major challenges in terms of maintenance: dams, harbour structures, bridges, ships, and aircrafts. Future offshore wind fields, situated in a very aggressive environment for materials, will also be subject to significant alteration of mechanisms. Given the consequences of failure, monitoring must be carried out throughout the lifetime of the structure. Maintenance is an essential element in ensuring a target level of safety. It is conditional because it depends on the results of the in-situ instrumentation; It must lead to inspect at the right time and with the best technique in a budget context, hence, the optimization is necessary. Non-destructive testing techniques (NDT) offer a great potential but are under exploited. Further uncertainties related to the material, the environment and the NDT are not taken into account because of the exciting gap between the effective mathematical models of optimization in uncertain contexts, and probabilistic degradation models. The main reason is that they were developed for the design of structures and not to integrate NDT measures or to be coupled with optimization methods. The thesis aims to propose a new maintenance strategy for structures degraded from NDT results through meta-models. Meta-models are models with low number of parameters based on physical expertise and probabilistic pertinence one hand, and on the degradation and durability indicators directly accessible from NDT on the other hand. The thesis proposes a modelling approach of multiphasic degradation phenomena of reinforced concrete based on a formulation of non-stationary state-dependant stochastic process

Key Words Probabilistic degradation and maintenance modelling, Condition-Based Maintenance, State-dependant stochastic processes, risk-management, decision-making, gamma process, non-stationary degradation, NDT.

